QP CODE: 19101457

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Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) EXAMINATION, MAY 2019

Fourth Semester

Core Course - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND LAPLACE TRANSFORMS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 Admission onwards

F05CDA24

Maximum Marks: 80 **Time: 3 Hours**

Part A

Answer any **ten** questions.

Each question carries **2** marks.

1. Write the component equation and simplified component equation for a plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$.
2. Define derivative for a vector-valued function $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$. When one can say that the curve traced by $\mathbf{r}$ is piecewise smooth?
3. Define the **gradient vector** of a function in the plane. Represent the **directional derivative** of a differentiable function in the plane as a dot product.
4. Interpret the physical meaning of negative divergence of a vector field representing the velocity of a gas flow in XY plane.
5. Define Oriented surface with example.
6. Find the curl of the vector field $F = x^2 i + xj + xyk$.
7. True or False: "Any set of n integers form a complete set of residues modulo n ".
8. Define $pseudoprime\ to\ the\ base\ a$.
9. State Wilson's theorem.
10. Find $\mathscr{L}^{-1}\left\{\frac{s+3}{(s-1)(s+2)}\right\}$.
11. State first shifting theorem for Laplace Transform.
12. Using convolution theorem, evaluate $\mathscr{L}^{-1}\left[\frac{1}{s(s-a)}\right]$.

(10×2=20)

Part B

Answer any **six** questions.

Each question carries **5** marks.

13. A helicopter is to fly directly from a helipad at the origin in the direction of the point $(1, 1, 1)$ at a speed of 60 ft/sec. What is the position of the helicopter after 10 sec?
14. Define the **tangent plane** and the **normal line** at a point on a smooth surface in space. Find the plane tangent to the surface $z = x\cos y - ye^x$ at $(0, 0, 0)$.

15. Integrate $f(x,y,z) = -\sqrt{x^2+z^2}$ over the circle $r(t) = (acost)j + (asint)k, \;\; 0 \leq t \leq 2\pi.$

16. Define the following with formula

a) work done by a force b) Flow integral.

17. a) What do you mean by parametrization of a surface.
b) Find the parametric equation of a cylinder of fixed height and radius.

18. Derive the congruence: $a^{13} \equiv a \pmod{3.7.13}$ for all a.

19. If the integer $n > 1$ has the prime factorization $n = p_1^{k_1} p_2^{k_2} \ldots . p_r^{k_r}$, then prove that $\phi(n) = n(1-\frac{1}{p_1})(1-\frac{1}{p_2})\ldots .(1-\frac{1}{p_r})$.

20. Using definition of Laplace Transform, prove that $\mathscr{L}(e^{at}) = \frac{1}{s-a}$. Hence evaluate $\mathscr{L}(\cos\omega t)$ and $\mathscr{L}(\sin\omega t)$.

21. Find $\mathscr{L}^{-1}\left\{\frac{1}{s^2(s^2+\omega^2)}\right\}$.

(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

22.
1. Define the **unit tangent vector, curvature** and **principal unit normal** for space curves.
2. Find the curvature and principal unit normal for the helix $\mathbf{r}(t) = (a\cos t)\mathbf{i} + (a\sin t)\mathbf{j} + bt\mathbf{k}, a, b \geq 0, a^2 + b^2 \neq 0.$

23. Verify Divergence Theorem for the field $F = xi + yj + zk$ over the sphere $x^2 + y^2 + z^2 = a^2$.

24.
1. State and prove Fermat's theorem.
2. Employ Fermat's theorem to prove that ,if p is an odd prime,then $1^p + 2^p + 3^p + \ldots . + (p-1)^p \equiv 0 \pmod{p}$.

25.
1. Let $f(t)$, $f'(t)$ be continuous and satisfy the growth restriction for all $t \geq 0$. Let $f''(t)$ be piecewise continuous on every finite interval on the semi-axis $t \geq 0$. Prove that the Laplace transform of $f''(t)$ satisfies $\mathscr{L}(f'') = s^2\mathscr{L}(f) - sf(0) - f'(0)$.
2. Solve the Initial value problem $y'' - 2y' - 3y = 0, \; y(1) = -3, \; y'(1) = -17$.

(2×15=30)