21100916

QP CODE: 21100916

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) EXAMINATION, MARCH 2021

Fourth Semester

Core Course - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND LAPLACE TRANSFORMS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 Admission onwards

2DC5189F

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Write the vector equation and the simplified component equation for a plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$.
2. Define continuity for a vector-valued function $\mathbf{r}(t)$. Also give an example for a continuous vector-valued function.
3. Define **torsion** of a smooth curve. Give a computational formula for the same.
4. Find the speed and velocity vector of the position vector $r(t) = (2t)i + (t^2)k,$. $0 \leq t \leq 2$.
5. Define gradient vector of a scalar function $f(x, y, z)$.
6. Find the parametrization of the cylinder of radius a and height h whose centre of the base is fixed on the origin of the XY plane.
7. State Fermat's theorem.
8. Derive the congruence : $a^{21} \equiv a \pmod{15}$ for all a.
9. Find the remainder when $15!$ is divided by 17.
10. Find the inverse laplace transform of the function $\frac{1}{(s-3)(s+5)}$.
11. Prove that the Laplace transform is a linear operation.

12. Let $f(t)$ be continuous and satisfies the growth restriction for all $t \geq 0$. Also let $f'(t)$ be piecewise continuous on every finite interval on the semi-axis $t \geq 0$. Prove that the Laplace transform of $f'(t)$ satisfies $\mathscr{L}(f') = s\mathscr{L}(f) - f(0)$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Define the length of a smooth curve $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k},\ a \leq t \leq b$. Also find the unit tangent vector of the curve $\mathbf{r}(t) = (3\cos t)\mathbf{i} + (3\sin t)\mathbf{j} + t^2\mathbf{k}$.

14. Define the **gradient vector** of a function in the plane. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.

15. Find the circulation density, and interpret its meaning for the vector fields
    1) $F(x, y) = -(cy)i + (cx)j$ where c is a constant
    2) $F(x, y) = yi$.

16. Find the flux of the field $F = yi - xj + k$ across the portion of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant in the direction away from the origin.

17. Find the divergence and curl of $F = (xyz)i + (3x^2y)j + (xz^2 - y^2z)k$ at $(1, 2, -1)$.

18. Show that 41 divides $2^{20} - 1$.

19. Prove: $\phi(n) = \frac{n}{2}$ if and only if $n = 2^k$ for some $k \geq 1$.

20. State and prove Existence theorem for Laplace Transforms.

21. Find $\mathscr{L}^{-1}\left\{\frac{2s-56}{s^2-4s-12}\right\}$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22.
    1. Define the **curvature** of a smooth plane curve. Find the curvature of a circle of radius a.
    2. Find and graph the **osculating circle** of the parabola $y = x^2$ at the origin.

23. State Divergence Theorem and use it to find the outward flux of the field

$F = x^2 i + y^2 j + z^2 k$ across the boundary of the cube cut from the first octant by the planes $x = 1, y = 1$ and $z = 1$.

24. 1. Prove: If n is an odd pseudoprime ,then $M_n = 2^n - 1$ is a larger one.
    2. Show that the integers 1105,2821,2465 are absolute pseudoprimes.

25. 1. State and prove convolution theorem.
    2. Using convolution theorem, solve $y'' + 3y' + 2y = 1, y(0) = y'(0) = 0.$

(2×15=30)