21102896

QP CODE: 21102896

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) EXAMINATIONS, OCTOBER 2021

Fourth Semester

Core Course - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND LAPLACE TRANSFORMS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2019 Admission only

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Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the component equation and simplified component equation for the plane through $P_0(-3, 0, 7)$ perpendicular to $\mathbf{n} = 5\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.
2. State and prove the **Dot Product Rule** for differentiating vector functions.
3. Define the tangential and normal scalar components of acceleration.
4. What is k - component of curl of a vector ? .
5. Write the equation of the surface area differential for a parametrized surface S.
6. Describe the one sided surface of Mobius band.
7. Check whether the following set S of integers constitute a complete set of residues modulo 7 or not:
   $S = \{-12, -4, 11, 13, 22, 82, 91\}$.
8. Define $pseudoprime$.
9. Prove: $\phi(3n) = 3\phi(n)$ if and only if $3|n$.
10. Using definition of Laplace Transform, prove that $\mathscr{L}(\sin \omega t) = \frac{\omega}{s^2+\omega^2}$.
11. Find $\mathscr{L}^{-1}\left\{\frac{1}{s(s^2+\omega^2)}\right\}$.

12. Define the convolution of the functions $f(t)$ and $g(t)$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Give the standard parametrization of the line through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$. Also write two different parametric equations for the line through $P(-3, 2, -3)$ and $Q(1, -1, 4)$.

14. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$.

15. Find the line integral of $f(x, y, z) = 2x + 3y + z$ over the straight line segment from $(0, 1, -1)$ to $(1, 1, 2)$.

16. Evaluate $\int(F.\,dr)$, where $F(x, y, z) = zi + xyj - y^2k$ along the curve $C; r(t) = t^2i + tj + \sqrt{t}k, 0 \le t \le 1$.

17. a) What is the component test for the exactness of the differential form $Mdx + Ndy + Pdz$.
    b) Is the field $F = (z + y)i + zj + (y + x)k$ conservative? Why? .

18. Prove: If p and q are distinct primes with $a^p \equiv a \pmod{q}$ and $a^q \equiv a \pmod{p}$ then $a^{pq} \equiv a \pmod{pq}$.

19. Derive the congruence: $a^7 \equiv a \pmod{42}$ for all a.

20. Let $f(t),\ f'(t)$ be continuous and satisfy the growth restriction for all $t \ge 0$. Also let $f''(t)$ be piecewise continuous on every finite interval on the semi-axis $t \ge 0$. Prove that the Laplace transform of $f''(t)$ satisfies $\mathscr{L}(f'') = s^2\mathscr{L}(f) - sf(0) - f'(0)$.

21. Solve $y'' - \frac{1}{4}y = 0,\ y(0) = 4,\ y'(0) = 0$ using Laplace Transform.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22.
    1. Find and graph the **osculating circle** of the parabola $y = x^2$ at the origin.
    2. Find the **curvature** for the helix $\mathbf{r}(t) = (a\cos t)\mathbf{i} + (a\sin t)\mathbf{j} + bt\mathbf{k}, a, b \ge 0, a^2 + b^2 \ne 0.$

23. Verify any one form of Green's Theorem for the vector field $F(x,y) = (x-y)i + xj$ and the region R bounded by the unit circle $C : r(t) = (cost)i + (sint)j$, $0 \leq t \leq 2\pi$.

24.
1. State and prove Wilson's theorem.
2. Prove the converse of Wilson's theorem.

25.
1. State and prove Convolution theorem.
2. Solve $y(t) - \int_0^t y(\tau)\sin(t-\tau)\,d\tau = \cos t$.

(2×15=30)