

QP CODE: 20100916



Reg No :

Name :

B.Sc DEGREE (CBCS) EXAMINATION, MARCH 2020

Fourth Semester

**Core Course - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND
LAPLACE TRANSFORMS**

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc
Mathematics Model II Computer Science)

2017 Admission onwards

1A6C9E8C

Time: 3 Hours

Marks: 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find parametric equations for the line through $(-2, 0, 4)$ parallel to $\mathbf{v} = 2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.
2. Define limit of a vector-valued function $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$.
3. Define the length of a smooth curve $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $a \leq t \leq b$. Also write the formula for length in terms of the length of a velocity vector $d\mathbf{r}/dt$.
4. Define flow integral along a curve.
5. Define divergence of a vector field.
6. State Divergence Theorem.
7. True or False: " If $ca \equiv cb \pmod{n}$ for some integer c then $a \equiv b \pmod{n}$ ". Give justifications.
8. Verify that $5^{38} \equiv 4 \pmod{11}$ using Fermat's theorem.
9. If $\gcd(a, 35) = 1$, show that $a^{12} \equiv 1 \pmod{35}$
10. Prove that the inverse Laplace Transform is a linear operation.
11. Find $\mathcal{L}(e^{-t} \sinh t)$.
12. Find $\mathcal{L}^{-1} \left\{ \frac{s-6}{(s-1)^2+4} \right\}$.

(10×2=20)





Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Define the **directional derivative** of a function in the plane. Using the definition, find the derivative of $f(x, y) = x^2 + xy$ at $P_0(1, 2)$ in the direction of the unit vector $\mathbf{u} = (1/\sqrt{2})\mathbf{i} + (1/\sqrt{2})\mathbf{j}$.
14. The cylinder $f(x, y, z) = x^2 + y^2 - 2 = 0$ and the plane $g(x, y, z) = x + z - 4 = 0$ meet in an ellipse E . Find parametric equations for the line tangent to E at the point $P_0(1, 1, 3)$.
15. Find the line integral of $f(x, y, z) = x + y + z$ over the straight line segment from $(1, 2, 3)$ to $(0, -1, 1)$.
16. Evaluate the line integral $\int (2x \cos y) dx - (x^2 \sin y) dy$ along the parabola $y = (x - 1)^2$ from $(1, 0)$ to $(0, 1)$.
17. Using spherical co-ordinate system, find the surface area of a sphere of radius a .
18. Let n be a composite square-free integer, say, $n = p_1 p_2 \dots p_r$, where the p_i are distinct primes.
If $p_i - 1 | (n - 1)$ for $i = 1, 2, \dots, r$, then prove that n is an absolute pseudoprime.
19. Let a, b, c are integers then prove that $\gcd(a, bc) = 1$ if and only if $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$.
20. If $\mathcal{L}(f(t)) = F(s)$ and c is any positive constant, show that $\mathcal{L}(f(ct)) = \frac{1}{c} F(\frac{s}{c})$.
21. Using convolution theorem, solve $y'' + y = \sin t$, $y(0) = 0$, $y'(0) = 0$.

(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

22.
 1. Define the **principal unit normal** vector for a smooth plane curve. Find the principal unit normal vector for the circular motion $\mathbf{r}(t) = (\cos 2t)\mathbf{i} + (\sin 2t)\mathbf{j}$.
 2. Find and graph the **osculating circle** of the parabola $y = x^2$ at the origin.
23. Verify Stokes Theorem for the hemisphere $S : x^2 + y^2 + z^2 = 9, z \geq 0$ its boundary circle $C : x^2 + y^2 = 9, z = 0$ and the field $F = yi - xj$.





24. 1. State and prove Wilson's theorem.
2. If p is a prime, prove that for any integer a , $p|a^p + (p-1)a$.
25. Using Laplace transform, solve $y(t) - \int_0^t y(\tau) d\tau = 1$.

(2×15=30)

