QP CODE: 19102030

19102030

Reg No : .....

Name : .....

**B.Sc. DEGREE (CBCS) EXAMINATION, OCTOBER 2019**

**Third Semester**

**COMPLEMENTARY COURSE - MM3CMT01 - MATHEMATICS - VECTOR CALCULUS, ANALYTIC GEOMETRY AND ABSTRACT ALGEBRA**

(Common to B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Chemistry Model III Petrochemicals, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III, B.Sc Geology and Water Management Model III, B.Sc Geology Model I, B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Physics Model III Electronic Equipment Maintenance)

2017 Admission Onwards

B98F064E

Maximum Marks: 80 Time: 3 Hours

**Part A**

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. State cross product rule for differentiation of vector functions.
2. Find the arc length parametrisation of the curve $r(t) = e^t\ cost\ i + e^t\ sint\ j + e^t k$ with base point at $t = 0$.
3. Define the directional derivative of a differentiable function on the plane.
4. Evaluate the line integral $\int_C (x + 3y)\,dx + (x - y)\,dy$ where $C : x = 2\cos t,\ y = 4\sin t,\ 0 \le t \le \frac{\pi}{4}$.
5. Define the flux of a three dimensional vector field $\mathbf{F}$ across an oriented surface $S$.
6. Define the divergence of a vector field in space.
7. Express the equation of the curve $x = 7$ in polar co-ordinates.
8. Find the equation of the ellipse with foci $(\pm\sqrt{2}, 0)$ and vertices $(\pm 2, 0)$.
9. Find the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$.
10. Find the number of generators of $Z_{11}$ under addition modulo 11.
11. What is the order of the Dihedral group $D_4$.
12. Find the number of elements in $\{f \in S_4 : f(1) = 1, f(2) = 2\}$.

(10×2=20)

## Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Find the equation of tangent to the ellipse $\frac{x^2}{4} + y^2 = 2$ at the point $(-2, 1)$..

14. Find the gradient of $f(x, y, z) = 3e^x cos(yz)$ at $(0, 0, 0)$and find the derivative of the function $f$ at this point in the direction of $n = 2i + j - 2k$.

15. Evaluate $\int_{(1,1,2)}^{(3,5,0)} yz\,dx + xz\,dy + xy\,dz$ .

16. Apply Green's Theorem to evaluate $\oint_C \left(e^x + y^2\right)\,dx + \left(e^y + x^2\right)\,dy$ where $C$ is the boundary of the region between $y = x^2$ and $y = 2x$ oriented counterclockwise.

17. Find the portion of the plane $y + 2z = 2$ inside the cylinder $x^2 + y^2 = 1$.

18. Find the focus, equation of the axis and the directrix of the parabola $y^2 = -2x$.

19. Find the vertices, focii, length of the semimajor axis and the length of the semiminor axis of the hyperbola $y^2 - 3x^2 = 3$.

20. Show that the set of all cube roots of unity forms a group under complex number multiplication.

21. Write all the elements and their order in the Group of all $2 \times 2$ matrices under matrix addition with addition modulo 2.

(6×5=30)

## Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a) Find the unit tangent, principal normal and curvature of the curve
$r(t) = acost\ i + asint\ j + btk$. $a,\ b \geq 0$ and $a^2 + b^2 \neq 0$.
(b) Find the directions in which $f(x, y, z) = x^3 - xy^2 - z$ increases most rapidly and decreases most rapidly at the point $(1, 1, 0)$.

23. Verify Stoke's Theorem for $\mathbf{F} = x^2 i + y^2 j + z^2 k$ where S is the portion of the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 1$ with upward orientation.

24. (a) Find the equation of the hyperbola when $16x^2 - 8y^2 = 16$ is shifted 1 units to the left and 3 units up. Also find the center,vertices, foci and directrix of the new hyperbola. Sketch the new hyperbola with all these details.
(b) Find the polar equation of the circle $(x - 4)^2 + (y - 3)^2 = 49$.

25. (a) Show by an example that every proper subgroup of a non abelian group may be abelian.
(b) How many homomorphisms are there from $Z$ to $Z$.

(2×15=30)