B.Sc DEGREE (CBCS)EXAMINATION, MARCH 2021

Third Semester

Complementary Course - MM3CMT01 - MATHEMATICS - VECTOR CALCULUS, ANALYTIC GEOMETRY AND ABSTRACT ALGEBRA

Common to B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Chemistry Model III Petrochemicals, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III, B.Sc Geology and Water Management Model III, B.Sc Geology Model I, B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Physics Model III Electronic Equipment Maintenance

2017 Admission Onwards

0388D6BB

Time: 3 Hours   Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the velocity and speed of the particle whose position is given by
   $r(t) = 2cost\ i + 3sint\ j + 4tk$ at $t = \frac{\pi}{2}$.

2. Define unit tangent vector of a curve.

3. Find the gradient of $f(x, y, z) = x^2 + y^2 + z^2$ at $(1, 1, 1)$.

4. Define the gradient field of a differentiable function. Find the gradient field of
   $f(x, y, z) = \ln \sqrt{x^2 + y^2 + z^2}$.

5. State the fundamental theorem of line integrals.

6. State normal form of Green's Theorem in the plane.

7. Express the equation of the curve $x^2 + y^2 = 4$ in polar co-ordinates.

8. Find the equation of the hyperbola with foci $(\pm\sqrt{2}, 0)$ and
   vertices $(\pm 2, 0)$ in standard form.

9. Find the eccentricity of the hyperbola $8x^2 - 2y^2 = 16$.

10. Define a cyclic group.

11. Give an example of a non abelian group so that all of its proper non trivial subgroups are cyclic.

12. Find the number of elements in $\{f \in S_4 / f(3) = 3\}$.

<div align="right">(10×2=20)</div>

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Find the derivative of $f(x, y) = 2x^2 + y^2$ at $(-1, 1)$ in the direction of the vector $u = 3i - 4j$.

14. Find the equation of tangent to curve $x^2 - y = 1$ at the point $(\sqrt{2}, 1)$.

15. Integrate $g(x, y, z) = x + y + z$ over the portion of the plane $2x + 2y + z = 2$ that lies in the first octant.

16. Using Stoke's Theorem, evaluate $\oint_C \mathbf{F} . d\mathbf{r}$ where $\mathbf{F} = xyi + yzj + zxk$ and C is the triangle $x + y + z = 1$ with vertices $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ with counterclockwise orientation looking from the first octant towards the origin.

17. Use Divergence Theorem to find the outward flux of $\mathbf{F} = x^2 i + y^2 j + z^2 k$ across the boundary of the cube bounded by the planes $x = \pm 1. y = \pm 1, z = \pm 1$.

18. Find the focus, equation of the axis and the directrix of the parabola $x = -3y^2$.

19. Find the standard equation of the hyperbola with focus $(4, 0)$ and directrix $x = 2$.

20. Show that a non trivial cyclic group with only one generator has exactly two elements.

21. Find all subgroups of Z_{18} and draw its subgroup diagram.

<div align="right">(6×5=30)</div>

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a) Find the unit tangent, principal normal and curvature of the curve
    $r(t) = a cos t\, i + a sin t\, j + bt k.\, a,\, b \geq 0$ and $a^2 + b^2 \neq 0$.
    (b) Find the directions in which $f(x, y) = x^2 + xy + y^2$ increases most rapidly and decreases most rapidly at the point $(-1, 1)$.

23. Verify Gauss Divergence Theorem for $\mathbf{F} = xyi + yzj + xzk$ where S is the surface of the cube bounded by the planes $x = 0,\, x = 2,\, y = 0,\, y = 2,\, z = 0,\, z = 2$.

24. (a) Find the equation of the hyperbola when $5y^2 - 4x^2 = 20$ is shifted 2 units to the left and 3 units up. Also find the center, vertices, foci and directrix of the new hyperbola. Sketch the new hyperbola with all these details.

(b) Find the polar equation of the parabola with focus $(0, 0)$ and directrix $r \cos\left(\theta - \dfrac{\pi}{2}\right) = 2$.

25. Show that the set of all permutations on a three element set forms a non abelian group under permutation multiplication and also draw its subgroup diagram.

$(2\times15=30)$