21101237

QP CODE: 21101237

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) EXAMINATION, APRIL 2021

Sixth Semester

CORE - MM6CRT01 - REAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc Computer Applications Model III Triple Main

2017 Admission Onwards

E6D35C1D

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Give an example of a function defind on R which is discontinuous only at x=0
2. Show that the continuous image of an open interval need not be an open interval.
3. Define Lipschitz function
4. Prove that if $f : I \to R$ is differentiable at $c \in I$, then f is continuous at c
5. Using the chain rule, find the derivative of $f^n(x)$, where $f : I \to R$ is differentiable and $n \geq 2, n \in N$
6. Check the validity of the following statement with proper reasoning 'Let f be a differentiable function on $A \subset R$, with $f'(x) = 0, \forall x$, then f is a constant'
7. How Riemann integral is related with areas of regions in $\mathbb{R}^2$.
8. Evaluate $\int\limits_0^1 \frac{1}{x^2+1} dx$.
9. Validate the statement "Riemann Integrable functions on an interval $[a, b]$ are continous on $[a, b]$"
10. Evaluate $lim(\frac{sinnx}{1+nx})$ for $x \epsilon R, x \geq 0$
11. Show that the sequence of functions $h_n(x) = \frac{x^2}{n}$ converge uniformly on $[0, 8]$.
12. Do the limit of a convergent sequence of differentiable functions on an interval [a, b] is differentiable, if not what condition will make the limit function differentiable?

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Define Dirichlet's function. Show that it is not continuous at any point of R

14. Let $A \subseteq R$, let f and g be continuous on A to R, and let $b \in R$.
(a) Show that the function f+g is continuous on A.
(b) If $h : A \to R$ is continuous on A and $h(x) \neq 0$ for $x \in A$, show that quotient f/h is continuous on A.

15. Let $I \subseteq R$ be an interval and let $f : I \to R$ be monotone on I. Then prove that the set of points $D \subseteq I$ at which f is discontinuous is a countable set .

16. Let $f : R \to R$ defined by $f(x) = \begin{cases} x^2, & x \; is \; rational \\ x, & x \; is \; irrational \end{cases}$, Prove that f is differentiable at $x = 0$

17. Using Mean value Theorem, Prove the following inequality $-x \leq \sin x \leq x, \forall x \geq 0$

18. State and Prove L' Hospitals's Rule I?

19. State and prove Squeeze Theorem for Riemann integrability.

20. Evaluate $\int\limits_1^4 \frac{cos\sqrt{t}}{\sqrt{t}} dt.$

21. State a sufficient condition to guarantee $\int_a^b f = lim \int_a^b f_n$. Show that $\int_1^2 e^{-nx^2} dx = 0.$

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a) Let I = [a,b] be a closed bounded interval and let $f : I \to R$ be continuous on I. Then prove that f has an absolute maximum and an absolute minimum on I .
(b) State and prove Preservation of intervals Theorem.

23. (a.) State and Prove L'Hospital's Rule II
(b.) Using this, find the following
(i.) $\lim_{x \to 0+} \dfrac{\log \sin x}{\log x}$
(ii.) $\lim_{x \to \infty} \dfrac{\log x}{x}$

24. (a) Let $f \in \mathcal{R}[a,b]$ and if $(\mathcal{P}_n)$ is any sequence of tagged partitions of $[a,b]$ such that $||\mathcal{P}_n|| \to 0$, prove that $\int_a^b f = lim_n S(f;\mathcal{P})$.

(b) Suppose that f is bounded on $[a,b]$ and that there exists two sequences of tagged partitions $(\mathcal{P}_n)$ and $(\mathcal{Q}_n)$ of $[a,b]$such that $||\mathcal{P}_n|| \to 0$ and $||\mathcal{Q}_n|| \to 0$, but such that $lim_n S(f;\mathcal{P}_n) \neq lim_n S(f;\mathcal{Q}_n)$. Show that $f \notin \mathcal{R}[a,b]$.

25. (a)State and prove the additivity theorem for Riemann integrable functions.

(b)Let $f \in \mathcal{R}[a,b]$ and if $[c,d] \subset [a,b]$, then the restriction of f to $[c,d]$ is in $\mathcal{R}[c,d]$.

(2×15=30)