20100567

QP CODE: 20100567

Reg No : ....

Name : ....

BSc DEGREE (CBCS) EXAMINATION, MARCH 2020

Sixth Semester

Core course - MM6CRT01 - REAL ANALYSIS

B.Sc Mathematics Model I,B.Sc Mathematics Model II Computer Science,B.Sc Computer Applications Model III Triple Main

2017 Admission Onwards

BFF2D188

Time: 3 Hours

Maximum Marks :80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Let $A \subseteq R$. If $f : A \to R$ is continuous on A, then prove that |f| is continuous on A .
2. Give an example of a discontinuous function defined on a closed interval C but not bounded on C
3. Define Monotone function.Show that such functions need not be continuous.
4. Let f, g are differentiable functions, then prove that $f - g$ is also differentiable?
5. Given that the function $f : R \to R$ defined by $f(x) = x^5 + 4x + 3$ is invertible and let g be its inverse. Find the value of $g'(8)$?
6. Define increasing function with a proper example?
7. Let $f, g : [a, b] \to \mathbb{R}$, if $\dot{\mathcal{P}}$ is a tagged partition of $[a, b]$ show that $S(f + g; \dot{\mathcal{P}}) = S(f; \dot{\mathcal{P}}) + S(g; \dot{\mathcal{P}})$
8. Give an example of a function which is Riemann integrable on an interval $[a, b]$ in$\mathbb{R}$ but not continous in $[a, b]$
9. Give an example of a function on $[0, 1]$ which is Riemann integrable but not continous.
10. Evaluate $lim x^2 e^{-nx}$.
11. Define uniform norm of a bounded function $\phi : A \to R$.State a necessary and sufficient condition for uniform convergence of a sequence of bounded functions (f_n) on $A \subseteq R$

12. If $a > 0$, show that $lim \int_a^{\pi} \frac{(sinnx)}{(nx)} dx = 0.$

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Define Thomae's function on $(0, \infty)$ and show that it is continuous precisely at the irrational points in $(0, \infty)$.

14. Define $g : R \rightarrow R$ by g(x) = 2x for x rational, and g(x) = x + 3 for x irrational . Find all points at which g is continuous .

15. State and prove Bolzano's Intermediate value theorem.

16. State and Prove the Chain rule of differentiation ?

17. State and prove the first derivative test for extrema?

18. Evaluate the limit $\lim_{x \to 0+} (\sin x)^x, x \in (0, \pi)$

19. If f is continous on [a, b] then the indefinite integral defined by $F(z) = \int\limits_a^z f \; \forall z \in [a, b]$ is differentiable on $[a, b]$and $F'(x) = f(x) \; \forall \in x[a, b]$.

20. Evaluate $\int\limits_1^4 \frac{\sqrt{1+\sqrt{t}}}{\sqrt{t}} dt.$

21. Check the uniform convergence of (g_n) on $\mathbb{R}$ where $g_n(x) = \frac{x^2+nx}{n}$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a) State and prove Continuous Extension Theorem.
(b) Let I be a closed bounded interval and let $f : I \rightarrow R$ be continuous on I . Then prove that f is uniformly continuous on I .

23. (a.) State and Prove L'Hospital's Rule I
(b.) Using this, find the following
(i.) $\lim_{x \to 0+} \dfrac{\tan x - x}{x^3}, x \in (0, \frac{\pi}{2})$
(ii.) $\lim_{x \to 0+} \dfrac{\log \cos x}{x}$

24. (a) Suppose that $f : [a, b] \to \mathbb{R}$ and that $f(x) = 0$, except for a finite number of ponits $c_1, c_2, \ldots\ldots. c_n$ in $[a, b]$. Prove that $f \in \mathcal{R}[a, b]$ and $\int\limits_a^b f = 0$.

(b) If $g \in \mathcal{R}[a, b]$ and if $f(x) = g(x)$ except for a finite number of ponts in $[a, b]$, prove that $f \in \mathcal{R}[a, b]$ and that $\int\limits_a^b f = \int\limits_a^b g$.

25. (a) State and prove the Cauchy Criterion for Riemann integrability of a function $f : [a, b] \to \mathbb{R}$.

(b) Check the Riemann integrability of Dirichlet function.

(2×15=30)