## Name :

## MSc DEGREE (CSS) EXAMINATION , JANUARY 2022 <br> Second Semester CORE - PH010202 - QUANTUM MECHANICS-I <br> M Sc PHYSICS,M.Sc.SPACE SCIENCE 2019 Admission Onwards <br> 47BD899A

Time: 3 Hours
Weightage: 30

## Part A (Short Answer Questions)

Answer any eight questions.
Weight 1 each.

1. Write a short note about measurement of observables in quantum mechanics.
2. Prove that the wave functions in the position space and momentum space are Fourier transforms of each other.
3. Write down the normalisation conditions for position and momentum eigenkets in three dimensions.
4. Write down the Schrodinger equation for the time evolution operator.
5. What are energy eigenkets? Why are they called stationary states?
6. Show that an operator representing a constant of motion have identical form in Schrodinger picture and Heisenberg picture.
7. Show that momentum is a constant of motion for a free particle.
8. Show that the $2 \pi$ rotated spin state differs from the original ket by a minus sign.
9. Evaluate $J_{+}|j, j\rangle$.
10. Write down the matrix representation of $J_{+}$in the $\{|j, m\rangle\}$ basis.
$(8 \times 1=8$ weightage $)$

## Part B (Short Essay/Problems)

Answer any six questions.
Weight 2 each.
11. The energy eigenvalues of the Hamiltonian of the system are given by $E_{n}=\frac{n^{2} \pi^{2} \hbar^{2}}{2 m a^{2}}, n=1,2,3, \ldots$. Find the expectation value of the Hamiltonian in a state represented by $|\alpha\rangle=\frac{1}{\sqrt{2}}\left(\left|E_{1}\right\rangle+\left|E_{2}\right\rangle\right)$, where the energy eigen kets $\left|E_{n}\right\rangle$ form an orthonormal set.
12. Prove that $\operatorname{Tr}(\alpha X+\beta Y)=\alpha \operatorname{Tr}(X)+\beta \operatorname{Tr}(Y)$.
13. Prove that $[A, B C]=[A, B] C+B[A, C] ; \quad[A B, C]=A[B, C]+[A, C] B$.
14. What is time evolution operator? Express the infinitesimal time evolution operator in terms of the Hamiltonian.
15. Explain the origin of natural line width of spectral lines.
16. From the infinitesimal rotation operator $\mathcal{D}(d \phi)$ obtain the finite rotation operator $\mathcal{D}(\phi)$.
17. From $\left\langle S_{k}\right\rangle=\frac{\hbar}{2} \chi^{\dagger} \sigma_{k} \chi$, where $\chi$ is a two-component spinor, obtain an explicit forms of Pauli matrics $\sigma_{k}$.
18. Show that for angular momentum operator $L_{x},\left\langle x^{\prime}\right| L_{x}|\alpha\rangle=-i \hbar\left(-\sin \phi \frac{\partial}{\partial \theta}-\cot \theta \cos \phi \frac{\partial}{\partial \phi}\right)\left\langle x^{\prime} \mid \alpha\right\rangle$ in the spherical polar coordinates.
$(6 \times 2=12$ weightage $)$

## Part C (Essay Type Questions)

Answer any two questions.

## Weight 5 each.

19. Discuss how the sequential Stern Gerlach experiments lead to the idea of a complex vector space.
20. Obtain the energy eigenvalues and eigen kets of a one-dimensional harmonic oscillator.
21. What are Clebsch-Gordon coefficients? Evaluate the Clebsch-Gordon coefficients for the addition of two angular momenta with $j_{1}=1$ and $j_{2}=1 / 2$.
22. Obtain the radial wave equation for a system moving under a central potential. Discuss the behaviour of the radial wave function near the origin .
