## B.Sc. DEGREE (CBCS) EXAMINATION, OCTOBER 2019

Third Semester

## COMPLEMENTARY COURSE - ST3CMT03 - STATISTICS - PROBABILITY DISTRIBUTIONS

(Common to B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Physics Model I)

2017 Admission Onwards
97DE51CB
Maximum Marks: 80

## Part A

Answer any ten questions.
Each question carries 2 marks.

1. Define $\mathrm{r}^{\text {th }}$ raw moment and $\mathrm{r}^{\text {th }}$ central moment in terms of expectation.
2. Find the characteristic function of $f(x)=a e^{-a x} ; x>0, a>0$.
3. Mention two examples of random variables following discrete uniform distribution.
4. Obtain the mgf of Bernoulli distribution.
5. Obtain the mgf of binomial distribution.
6. If X and Y are independent Poisson random variables with parameters $\mathrm{m}_{1}$ and $\mathrm{m}_{2}$ respectively, show that $\mathrm{Z}=\mathrm{X}-\mathrm{Y}$ does not follow Poisson distribution.
7. Define one parameter gamma distribution.
8. Find the mean of two parameter gamma distribution.
9. Obtain the second raw moment of type - 1 beta distribution.
10. State Lindberg- Levy form of central limit theorem.
11. Mention any two uses of standard error.
12. Define student's $t$ distribution.

## Part B

Answer any six questions.
Each question carries 5 marks.
13. State and prove Cauchy - Schwartz inequality.
14. Let the joint pdf be $f(x, y)=2\left(x+y-3 x y^{2}\right) ; 0<x<1,0<y<1$. Find $E(X)$ and $E(Y)$.
15. For uniform distribution over ( $0, b$ ), find coefficient of variation.
16. Find the mean and variance of hyper geometric distribution.
17. Establish the lack of memory property of exponential distribution.
18. X is a normal random variable with mean 20 and SD 5 . Find the probability that (1) $16<\mathrm{X}<22$
(2) $\mathrm{X}>23$ (3) $1 \mathrm{X}-201>5$.
19. Show that the weak law of large numbers is true for the mean of a random sample of size $n$ from a population with finite mean and variance.
20. Derive the mgf of chi - square distribution and hence find mean and variance.
21. If X is a random variable following F distribution with $\left(\mathrm{n}_{1}, \mathrm{n}_{2}\right)$ degrees of freedom, show that $\mathrm{Y}=$ $1 / \mathrm{X}$ follows F distribution with $\left(\mathrm{n}_{2}, \mathrm{n}_{1}\right)$ degrees of freedom.

## Part C

Answer any two questions.
Each question carries 15 marks.
22. The joint pdf is given by $f(x, y)=2-x-y ; 0<x<1,0<y<1$ and 0 elsewhere. Find (1) $V(X)$ (2) V(Y) (3) COV( X, Y).
23. (a) Establish the lack of memory property of geometric distribution.
(b) Let X and Y be two independent random variables such that $\mathrm{P}(\mathrm{X}=\mathrm{r})=\mathrm{P}(\mathrm{Y}=\mathrm{r})=\mathrm{q}^{\mathrm{r}} \mathrm{p} ; \mathrm{r}=0,1$, $2, \ldots$ where $\mathrm{p}+\mathrm{q}=1$. Find the conditional distribution of X given $\mathrm{X}+\mathrm{Y}$.
24. (a) Obtain the mean, variance and harmonic mean of type -1 beta distribution.
(b) Show that type -2 beta distribution can be obtained from type -1 beta distribution using transformation of variables.
25. (1) State and prove Tchebycheff's inequality.
(2) Two unbiased dice are thrown and $X$ denotes the sum of the numbers shown. Find an upper bound to the probability that X will not be between 4 and 10 using Tchebycheff's inequality.
$(2 \times 15=30)$

