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Reg. No..............................

Name..................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2018

Third Semester

Faculty of Science

Branch I (A)–Mathematics

MT 03 C 15—OPTIMIZATION TECHNIQUES

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. How does integer programming differ from linear programming ?
2. What is the effect of the 'integer' restriction of all the variables on the feasible space of integer programming problem ?
3. Define the following :
   - (a) Competitive game.
   - (b) Pure and mixed strategies.
4. Define the following :
   - (a) Pay-off matrix.
   - (b) Rectangular game.
5. Define the following :
   - (a) A chain.
   - (b) A path.
   - (c) A connected graph.
6. Write a short note on sensitivity analysis.
7. Define the following terms :
   - (a) Gradient vector.
   - (b) Hessian matrix.
8. What are the primary uses of Kuhn–Tucker necessary and sufficient conditions ?

(5 × 1 = 5)

Turn over

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Describe the branch and bound method in integer programming.

10. Solve the problem using cutting plane method :

Maximise $x_1 + x_2$

subject to $7x_1 - 6x_2 \le 5$

$6x_1 + 3x_2 \ge 7$

$-3x_1 + 8x_2 \le 6$

x_1, x_2 non-negative integers.

11. Five villages in a hilly region are connected by roads. The direct distance (in km) between each pair of villages along a possible road and it cost of construction per km (in 10^4 rupees) are given below. Find the minimum cost at which all the villages can be connected and the roads which should be constructed :

		Distances				
		1	2	3	4	5
	1		18	12	15	10
Costs	2	3		15	8	22
	3	4	3		6	20
	4	5	5	6		7
	5	2	2	5	7	

12. Write a short note on sensitivity analysis.

13. Use Graphical methods to solve the game with pay-off matrix :

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \end{bmatrix}.$$

14. Explain the principle of dominance in Game theory.

15. Minimize the following objective function using a Golden section search. Use a resolution of $\in = 0.10$.

Minimize $f(x) = 3x^4 + (x-1)^2$

$4 \geq x \geq 0.$

16. Show that the point $x = (0.0)$ is a global minimum solution to $f(x) = x_2^2 + 3x_1^6 + 5x_2^4$.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) Maximize $5x_1 + 2x_2 + x_3$

subject to $x_1 + x_2 + 2x_3 \leq 10, \left| -3x_1 + 10x_2 - x_3 \right| \geq 15, x_1, x_2, x_3 \geq 0.$

(b) Express the following conditions as simultaneous constraints using 0.1 variables :

(i) Either $x_1 + 2x_2 \leq 4$ or $2x_1 + 3x_2 \geq 12$.

(ii) If $x_3 \leq 4$ then $x_4 \geq 5$, otherwise $x_4 \leq 2$.

(iii) $x_5 = 1$ or 3 or 5 only.

(iv) At least two of the following constraints are satisfied $x_6 + x_7 \leq 3, x_6 \leq 2, x_7 \leq 4, x_6 + x_7 \geq 5$.

18. State and prove Mini-max theorem.

19. Solve the following LP problem :

Maximize $Z = 3x_1 + 5x_2$

subject to the constraints $3x_1 + 2x_2 \leq 18$

$x_1 \leq 4$

$x_2 \leq 6, x_1, x_2 \geq 0$

and (a) Determine the optimal solution of the problem ; (b) Discuss the change in c_j on the optimality of the optimal basic feasible solution.

20. Write both the primal and dual L_P problems corresponding to the rectangular game with the following pay-off matrix :

$$\begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix}.$$

Solve the game by solving L_P problems by simplex method.

Turn over

21. Solve the following problem using only the Kuhn–Tucker conditions :

$f(x) = 100 - 1.2x_1 - 1.5x_2 + 0.3x_1^2 + 0.05x_2^2$

subject to $g_1(x) = x_1 + x_2 \geq 35$

$g_2(x) = x_1 \geq 0$

$g_3(x) = x_2 \geq 0.$

22. Use projected gradient method to solve the problem :

Minimize $f(x) = (x_1 - 3)^2 + (x_2 - 4)^2.$

subject to $2x_1 + x_2 = 3.$

(3 × 5 = 15)