QP CODE: 21000383

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Reg No :

Name :

M Sc DEGREE (CSS) EXAMINATION, MARCH 2021

Third Semester

Faculty of Science

CORE - ME010303 - MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

167C3C33

Time: 3 Hours

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find the Fourier Series for $f(x) = 8x, 0 < x < 2\pi$
2. State Fourier Integral theorem and its exponential form.
3. Show that the existence of partial derivatives at any point, does not imply the existence of directional derivative thereat.
4. Define total derivative and Jacobian matrix.
5. Give matrix representation for a linear function $T : \mathbf{R}^n \to \mathbf{R}^m$.
6. Define Jacobian determinant and find the Jacobian determinant for the function $f(z) = 9z + 2z^2$
7. For some integer $n \geq 1$, let f have a continuous n^{th} derivative in the open interval (a, b). Also for some interior point c in (a, b), $f'(c) = f''(c) = \ldots\ldots$ but $f^n(c) \neq 0$ If n is even and $f^n(c) < 0$ then prove that f has a local maximum at c
8. Define a Stationary point and a Saddle point.
9. $Let\ G(x) = \sum_{i \neq m} x_i e_i + g(x) e_m,\ x \in E\ be\ a\ primitive\ mapping\ and\ (D_m g)(a) \neq 0.\ Prove\ that\ G^{'}(a)\ is\ invertible.$
10. Define k- forms and explain its elementary properties.

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. State and prove Weierstrass Approxiamation Theorem for real valued and continuous functions on compact interval.
12. Let $R = (-\infty, \infty)$ and $f \in L(R), g \in L(R)$ and that either f or g is bounded on R. prove that the convolution integral $h(x) = \int_{-\infty}^{\infty} f(t) g$ for every x in R. Moreover if the bounded function f or g is continuous on R. Then h is also continuous on R and $h \in L(R)$.
13. Compute the gradient vector $\nabla f(x, y)$ at those points $(x, y) \in \mathbf{R}^2$ if
    a. $f(x, y) = x^2 y^2 log(x^2 + y^2)\ if\ (x, y) \neq (0, 0), f(0, 0) = 0$.
    b. $f(x, y) = xysin(x^2 + y^2)$
14. a. State and prove mean value theorem.
    b. If f is a real valued function on an open convex subset S of $\mathbf{R}^n$ prove that $f(\mathbf{y}) - f(\mathbf{x}) = \nabla f(\mathbf{z}).(\mathbf{y} - \mathbf{x})$ for $\mathbf{z}$ in the line segment joining
15. (a) Let A be an open subset of R^n and assume that $f : A \to R^n$ is continuous and has finite partial derivatives $D_j f_i$ on A. If f is one-to-one on A and if
    x in A then prove that $f(A)$ is open.
    (b) Let A be an open subset of R^n and assume that $f : A \to R^n$ has continuous partial derivatives $D_j f_i$ on A. If $J_f(x) \neq 0$ for all x in A then prove
    mapping .

16. Assume that the second order partial derivatives $D_{i,j}f$ exist in an n- ball $B(a)$and are continuous at a , where $a$is a stationary point of f.

Let $Q(t) = \frac{1}{2}f''(a;t) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} D_{i,j}f(a)t_it_j$.Prove the following

(a) If $Q(t) > 0$ for all $t \neq 0$, then f has a relative minimum at a.

(b) If $Q(t) < 0$ for all $t \neq 0$, then f has a relative maximum at a .

(c) If $Q(t)$ takes both positive and negative values , then $f$has a saddle point at a .

17. $For\ every\ f \in C(I^k)\ prove\ that\ the\ order\ of\ integration\ is\ immaterial\ for\ \int_{I^k} f(x)dx.$

18. $If\ \Phi(r,\theta,\phi) = (x,y,z)\ where\ x = rsin\theta cos\phi,\ y = rsin\theta sin\phi,\ z = rcos\theta\ and\ D\ is\ the\ 3-cell\ defined\ by\ 0 \leq r \leq 1,\ 0 \leq \theta \leq$

$then\ show\ that\ \int_{\Phi} dx \wedge dy \wedge dz = \frac{4\pi}{3}.$

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. State and prove the Convolution Theorem for Fourier Transforms.

20. Show that if $\mathbf{g}$ is differentiable at $\mathbf{a}$ and $\mathbf{f}$ is differentiable at $\mathbf{b} = \mathbf{g}(\mathbf{a})$, then the composition function, $\mathbf{h} = \mathbf{f}\ o\ \mathbf{g}$ is differentiable at $\mathbf{a}$ and $\mathbf{h}'(\mathbf{a}) = \mathbf{f}'(\mathbf{b})\ o\ \mathbf{g}'(\mathbf{a})$. Also, express this result in matrix form.

21. Assume that one of the partial derivatives $D_1f, D_2f, \dots. D_nf$ exist at c and the remaining $n-1$ partial derivatives exist in some n- ball $B(c)$ and are co

Prove that f is differentiable at c

22. $State\ the\ partitions\ of\ unity\ theorem.\ Prove\ that\ if\ f \in C(R^n)\ and\ the\ support\ of\ f\ lies\ in\ K,$

$then\ f = \sum_{i=1}^{s} \psi_i f,\ where\ each\ \psi_i f\ has\ its\ support\ in\ some\ V_\alpha.$