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Reg. No..............................

Name..................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2018

Third Semester

Faculty of Science

Branch I (A) : Mathematics

MT 03 C11—MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. If $f \in L([0, p])$ and if f has period p, write the Fourier series generated by f.

2. Write the exponential form of Fourier integral theorem.

3. If $f : R^2 \to R$ defined by

$$f(x, y) = \begin{cases} x + y & \text{if } x = 0 \text{ or } y = 0 \\ 1 & \text{otherwise} \end{cases}.$$

Find $D_1 f(0, 0)$ and $D_2 f(0, 0)$.

4. Write the first order Taylor formula for a function $f : S \to R^m$ which is differentiable at a point c.

5. State inverse function theorem.

6. If $f(x, y) = x^4 + y^4 - 4x^2y^2$. Verify that the mixed partial derivatives $D_{1,2} f$ and $D_{2,1} f$ are equal.

7. State Stoke's theorem.

8. Define a differential k-form.

$(5 \times 1 = 5)$

Turn over

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. State and prove Weierstrass approximation theorem.

10. Show that if $p > 0$ and $q > 0$.

$$\int_0^1 x^{p-1}(1-x)^{q-1}\,dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}.$$

11. Assume that f is differentiable at c with total derivative T_c. Prove that the directional derivative $f'(c, n)$ exists for every n in R^n and $T_c(n) = f'(c, n)$.

12. Calculate all first-order partial derivatives and the directional derivative $f'(x, n)$ for the function $f(x) = a \cdot x$ where a is a fixed vector in R^n.

13. Let S be an open connected subset of R^n and let $f : S \to R^m$ be differentiable at each point of S. If $f'(c) = 0$ for each $c \in S$, prove that f is constant on S.

14. Find and classify the extremum values (if any) of $f(x, y) = x^2 + y^2 = x + y + xy$.

15. If ω and λ are K-and M-forms, respectively of class ξ' in E, show that $d(\omega \wedge \lambda) = (d\omega) \wedge \lambda + (-1)^k \omega \wedge d\lambda$.

16. If $r(t) = (a\cos t, b\sin t)\ 0 \le t \le 2\pi$. Find $\int_r xdy$ and $\int_r ydx$

(5 × 2 = 10)

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Part C

Answer any **three** *questions.*
Each question has weight 5.

17. State and prove convolution theorem for Fourier transforms.

18. State and prove chain rule of differentiation.

19. (a) Compute the gradient vector $\nabla f(x, y)$ at those points (x, y) in R^2 where it exists for the function $f(x, y) = xy \sin\left(\frac{1}{x^2 + y^2}\right)$ if $(x, y) \neq (0, 0), f(0, 0) = 0$.

    (b) Let $f : R^2 \to R^3$ defined by $f(x, y) = (\sin x \cos y, \sin x \sin y, \cos x \cos y)$. Determine the Jacobian matrix $Df(x, y)$.

20. Assume that one of the partial derivatives $D_1 f, D_2 f \ldots\ldots D_n f$ exists at c and the remaining $n-1$ partial derivatives exist in some $n-$ball $B(c)$ and are continuous at C. Prove that f is differentiable at C.

21. Let A be an open subset of R^n and assume that $f : A \to R^n$ has continuous partial derivatives $D_j f_i$ on A. If $J_r(x) \neq 0$ for all $x \in A$, prove that f is an open mapping.

22. Let E be an open set in R^n, T is a ξ' mapping of E into an open set $V \subset R^m$ and ω and λ be k and m forms in V respectively, prove that :

    (a) $(\omega + \lambda)_T = {}^{\omega}T + {}^{\lambda}T$ if $k = m$.

    (b) $(\omega \wedge \lambda)_T = {}^{\omega}T \wedge {}^{\lambda}T$.

    (c) $d\left({}^{\omega}T\right) = (d\omega)_T$ if ω is of class ξ' and T is of class ξ^n.

(3 × 5 = 15)