QP CODE: 21101715

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) SPECIAL SUPPLEMENTARY EXAMINATION, JULY 2021

Fifth Semester

CORE COURSE - MM5CRT01 - MATHEMATICAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc Computer Applications Model III Triple Main

2018 Admission Only

EA4204A8

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that IfA is infinite and B is finite, then $A - B$ is infinite?
2. Determine the set $A = \{(x, y) \in R \times R : |x| = |y|\}$?
3. If $t > 0$ prove that there exist an $n_t \in N$ such that $0 < \dfrac{1}{n_t} < t$
4. Define various types of intervals with proper examples?
5. Prove that a sequence in R can have at most one limit point.
6. Find $lim(\frac{(-1)^n n}{n^2+1})$.
7. Let X = (2, 4, 6,..., 2n,...) and Y = $\left(1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\right)$. Find X - Y and X/Y.
8. Give an example of an unbounded sequence that has a convergent subsequence. Explain.
9. Prove that a monotone sequence of real numbers is properly divergent if and only if it is bounded.
10. State Integral Test.
11. Define an Alternating Series. Give an example.
12. Define the limit of a function f at c, Where $f : A \to \mathscr{R}$ and c is a cluster point of A.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Let S be a non empty subset of real numbers which is bounded above .Define $a+S=\{a+s: s \in S\}$, Prove that $Sup(a+S)=a+Sup\ S$

14. Using Cantor's diagonal argument, prove that the unit interval $[0,1]$ is not countable?

15. Let X = (x_n) be a sequence that converges to x. Prove that the sequence of absolute values ($|x_n|$) converges to |x|.

16. Prove that for any real number a > 0, there exists a sequence (s_n) of real numbers that converges to $\sqrt{a}$.

17. Let X = (x_n) be the sequence defined as x_1 = 1, x_2 = 2 and $x_n=\frac{x_{n-2}+x_{n-1}}{2}$ for n > 2. Prove that lim X = $\frac{5}{3}$.

18. Show that if a series contans only a finite number of negative terms, then it is absolutely convergent.

19. Test the convergence and absolute convergence of the series whose nth term is $\frac{n^n}{(n+1)^{n+1}}$

20. Evaluate the one-sided limits of the function $h(x)=\frac{1}{(e^{\frac{1}{x}}+1)}$ at $x=0$.

21. Give an example of a function that has a right-hand limit but not a left-hand limit at a point.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Prove that the set of all real numbers is a complete ordered field?

23. (a) State and prove Monotone Convergence Theorem.
(b) Let x_1 = 2 and x_{n+1} = 2 + $\frac{1}{x_n}$. Prove that lim(x_n) = 1 + $\sqrt{2}$.

24. (a) State and prove Comparison Test for the convergence of series. (b) Discuss the convergence of

    - $\sum_1^\infty \frac{1}{n^2+n}$
    - $\sum_1^\infty \frac{1}{n!}$

25. (a) Let $A \subseteq \mathscr{R}$, $f : A \to \mathscr{R}$ and let $c \in \mathscr{R}$ be a cluster point of A. If $a \leq f(x) \leq b$ for all $x \in A$,$x \neq c$, and if $\lim\limits_{x \to c} f$ exists,Then prove that $a \leq \lim\limits_{x \to c} f \leq b$.

(b) Check whether the following limits exist or not. Give explanations

(1) $\lim\limits_{x \to 0}(\frac{sinx}{x})$ (2) $\lim\limits_{x \to 0}(xsin(\frac{1}{x}))$

(2×15=30)