19102565

QP CODE: 19102565

Reg No : ....

Name : ....

BSc DEGREE (CBCS) EXAMINATION, OCTOBER 2019

Fifth Semester

Core Course - MM5CRT01 - MATHEMATICAL ANALYSIS

B.Sc Computer Applications Model III Triple Main ,B.Sc Mathematics Model I,B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

819A8253

Maximum Marks: 80 Time: 3 Hours

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Let $S = \{1, 2\}$ and $T = \{a, b, c\}$ then determine all the injections from S to T ?
2. Define the notion of inequlity between two real numbers in terms of the positive set P in the ordering property of real numbers?
3. Define bounded and unbounded sets with proper examples?
4. Define rational numbers in terms of the decimal expansion? Find the decimal representation of $-\dfrac{2}{7}$?
5. If 0 < b < 1, prove that $lim(b^n) = 0$.
6. Define bounded sequence. Give an example.
7. Find the limit of $(3n^{1/2n})$.
8. Define Cauchy sequence. Give an example.
9. Let (x_n) and (y_n) be two sequences of real numbers and suppose that $x_n \leq y_n$ for all n. Prove that if $\lim x_n = +\infty$ then $\lim y_n = +\infty$.
10. State the Limit Comparison Test for the series.
11. Is the series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ is absolutely convergent or not? Why?
12. Show that $\lim_{x \to c} x = c$ for any $c \in \mathscr{R}$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Show that for all $a, b \in R$

    (a.) $\max\{a,b\} = \dfrac{1}{2}(a+b+|a-b|)$

    (b.) $\min\{a,b\} = \dfrac{1}{2}(a+b-|a-b|)$

14. Prove that If A, B are bounded sets then $Sup\ (A+B) = Sup\ A + Sup\ B$ where $A+B=\{a+b : a \in A, b \in B\}$

15. Prove that lim $(n^{1/n}) = 1$.

16. Prove that $lim(\frac{sinn}{n}) = 0$.

17. State and prove Monotone Convergence Theorem.

18. State and prove the root test for the absolute convergence of a series in R.

19. State and prove Abel's Lemma.

20. Prove that $\lim\limits_{x\to 0} \cos(\frac{1}{x})$ does not exist but that $\lim\limits_{x\to 0} x\cos(\frac{1}{x}) = 0$.

21. Check whether the one-sided limits of the function $g(x) = e^{\frac{1}{x}}$ at $x = 0$ exist or not.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a.) State and Prove Nested interval property?

    (b.) Prove that the set of real numbers is not countable?

23. (a) State Monotone Convergence Theorem.

    (b) Prove that for any real number a > 0, there exists a sequence (s_n) of real numbers that converges to $\sqrt{a}$.

    (c) Use the above sequence to evaluate the value of $\sqrt{5}$ correct to 5 decimal places.

24. Test the convergence and absolute convergence of the following series.

    - $\sum_1^\infty \frac{(-1)^{n+1}}{(n+1)}$
    - Whose nth term is $\frac{(-1)^n n^n}{(n+1)^{n+1}}$

25. (a) Let $A \subseteq \mathscr{R}$, $f, g : A \to \mathscr{R}$, and let $c \in \mathscr{R}$ be a cluster point of A, Suppose that $f(x) \le g(x)$ for all $x \in A$, $x \neq c$, Then prove the following

- If $\lim_{x \to c} f = \infty$, then $\lim_{x \to c} g = \infty$.
- If $\lim_{x \to c} g = -\infty$,then $\lim_{x \to c} f = -\infty$.

(b) Give an example of a function that has a right-hand limit but not a left-hand limit at a point.

(c) Evaluate the limit or show that it do not exist " $\lim_{x \to 1} \frac{x}{x-1}$ where $x \neq 1$.

(2×15=30)