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## M.Sc. DEGREE (C.S.S.) EXAMINATION, DECEMBER 2018

## First Semester

## Faculty of Science

Branch II : Physics-(A)-Pure Physics
PH 1C 01—MATHEMATICAL METHODS IN PHYSICS-I
(2012 Admission onwards)
Time : Three Hours
Maximum Weight : 30

## Part A

Answer any six questions.
Each question carries 1 weight.

1. Explain the characteristics of irrotational and solenoidal vectors.
2. State and prove Gauss' divergence theorem.
3. Write note on Hilbert space.
4. What is similarity transformation ? How does it affect the eigen values and eigen vectors of a matrix?
5. State and prove Cayley - Hamilton theorem for square matrices.
6. Prove that any two eigen vectors of a real symmetric matrix are orthogonal provided that the corresponding eigen values are different.
7. Explain the central limit theorem.
8. Show that Kronecker delta is a mixed tensor of rank 2.
9. Show that $\mathrm{H}_{2 n}(0)=(-1)^{n} \frac{(2 n)!}{n!}$.
10. Show that $\mathrm{J}_{n}(-x)=(-1)^{n} \mathrm{~J}_{n}(x)$.

## Part B

Answer any four questions.
Each question carries 2 weight.
11. Show that $\hat{\mathrm{P}}(x)=-i \hbar \frac{\partial}{\partial x}$ is Hermitian.

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12. Solve the system of equations

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\begin{array}{r}
x+y+3 z=6 \\
2 x+3 y-4 z=6 \\
3 x+2 y+7 z=0
\end{array}
$$

by Gauss elimination method.
13. Show that contraction reduces the rank of a tensor by 2 .
14. Four persons are chosen at random from a group of 3 men, 2 women and 4 children. Find the probability that exactly two of them were children.
15. Prove that $\beta\left(\frac{1}{2}, \frac{1}{2}\right)=\pi$.
16. Construct a scalar from the tensor $\mathrm{A}_{k l}^{i j}$.

## Part C

Answer all questions.
Each question carries 4 weight.
17. (a) Define general orthogonal curvilinear co-ordinates and obtain various differential operators in terms of orthogonal curvilinear co-ordinates.

## Or

(b) Obtain the equation of continuity and the equation of motion for the flow of an incompressible fluid using vector methods and prove that the condition of incompressibility of fluid motion is div $v=0$, where $v$ is the velocity vector.
18. (a) Write notes on (i) Riemann - Christoffel tensor ; and (ii) Ricci tensor.

## Or

(b) What are recurrence relations? State and prove any three recurrence formulae for Hermite polynomials.
19. (a) What are unitary and Hermitian matrices? State and prove their properties.

Or
(b) Outline the characteristics of Binomial, Poisson and Gaussian distributions.
20. (a) Define Legendre polynomials and prove their orthogonality condition.

Or
(b) Find the solution of Bessel's differential equation using Frobenius method.

