21101240

QP CODE: 21101240

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) EXAMINATION, APRIL 2021

Sixth Semester

CORE - MM6CRT04 - LINEAR ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

2D568F24

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Write 3x3 matrix whose entries are given by $x_{ij} = i + j$
2. *Prove that a real 2x2 matrix is orthogonal if and only if it is of one of the forms* $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ *where* $a^2 + b^2 = 1$
3. If V is a vector space over a field F. Prove that a) If $\lambda x = 0$ then either $\lambda = 0$ or $x = 0$ b) $\forall x \in V, \forall \lambda \in F$ $(-\lambda)x = -(\lambda x) = \lambda(-x)$
4. *Define span S of a vector space V and Prove that* $\{e_1, e_2, \dots. e_n\}$ *is a spanning set of* R^n.
5. Prove that { (1,1,1), (1,2,3) , (2,-1, 1) } is a basis of R^3.
6. If $f : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $f(a, b) = (b, 0)$, prove that $Im\, f = Ker\, f$.
7. Define surjective and injective linear mappings.
8. Determine the transition matrix from the ordered basis $\{(1, -1, 1), (1, -2, 2), (1, -2, 1)\}$ of $\mathbb{R}^3$ to the natural ordered basis of $\mathbb{R}^3$.
9. Define a nilpotent linear mapping f on a vector space V of dimension n over a field F. What is meant by index of nilpotency of f.
10. Find the eigen values of A = $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$
11. Define the eigen space and geometric multiplicity associated with the eigen value.
12. Define diagonalizable linear map and diagonalizable matrix.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. *Reduce to Hermite form* $\begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 2 & 4 & 4 & 8 & 4 \\ 3 & 6 & 5 & 7 & 7 \end{bmatrix}$

14. Determine whether or not the matrices are row equivalent. $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & -1 \\ -3 & -2 & 3 \end{bmatrix}$

15. a)Show that the matrix $\begin{bmatrix} 1 & 3 & 4 & 7 \\ 2 & 3 & 5 & 8 \\ 1 & 4 & 5 & 9 \end{bmatrix}$ has neither a left inverse nor a right inverse.

    b)*Prove that if A and B are invertible matrix , then* $(AB)^{-1} = B^{-1}A^{-1}$

16. *Prove that the set of lower triangular nxn matrices is a subspace of the vector space*

17. a) Define rank and nullity of a linear mapping. Find the rank and nullity of $pr_1 : \mathbb{R}^3 \to \mathbb{R}$ defined by $pr_1(x, y, z) = x$.

    b) Let V and W be vector spaces each of dimension n over a field F. If $f : V \to W$ is linear, then prove that f is surjective if and only if f is bijective.

18. Prove that a square matrix is invertible if and only if it represents an isomorphism.

19. Define similar matrices. Prove that for every $\vartheta \in \mathbb{R}$, the complex matrices $\begin{bmatrix} cos\,\vartheta & -sin\,\vartheta \\ sin\,\vartheta & cos\,\vartheta \end{bmatrix}$, $\begin{bmatrix} e^{i\vartheta} & 0 \\ 0 & e^{-i\vartheta} \end{bmatrix}$ are similar.

20. Find the eigen values and a basis of each of the corresponding eigen space $\begin{bmatrix} -7 & 1 & -1 \\ -7 & 1 & -1 \\ -6 & 6 & -6 \end{bmatrix}$

21. For the nXn tridiagonal matrix An = $\begin{bmatrix} 2 & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 2 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 2 & 1 & \dots & 0 & 0 \\ . & . & . & . & \dots\dots & . & . \\ 0 & 0 & 0 & 0 & \dots & 2 & 1 \\ 0 & 0 & 0 & 0 & \dots & 1 & 2 \end{bmatrix}$ Prove that det An = n + 1.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a)Find the row rank of the matrix $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & -1 & 1 \\ 5 & 4 & -2 & 2 \end{bmatrix}$

    b)Prove that the rows $x_1, x_2, \ldots, x_p$ where $p \geq 2$ are linearly dependent if and only if one of the x_i can be expressed as a linear combination of the other.

    c)Prove that elementary row operations do not affect row rank.

23. a)Prove that the set W of complex matrices of the form $\begin{bmatrix} \alpha & \beta \\ -\bar{\beta} & -\alpha \end{bmatrix}$ is a real vector space of dimension 4.

b)If V is a vector space over C of dimension n, prove that V is a vector space over R of dimension 2n .

c) If S is a subset of V then prove that S is a basis if and only if S is a maximal independent subset.

24. a) Define linear mapping from a vector space to a vector space. If $f : V \to W$ is linear, then define direct image of X under f and inverse image of Y under f, for every subset X of V and for every subset Y of W.

b) Prove that direct image of X under f and inverse image of Y under f, are inclusion-preserving.

c) Prove that direct image of X under f and inverse image of Y under f, carries subspaces to subspaces.

25. Show that $\{(1,1,0),(1,0,1),(0,1,1)\}$ is a basis of $\mathbb{R}^3$. If $f : \mathbb{R}^3 \to \mathbb{R}^2$ is linear and such that $f(1,1,0) = (1,2), \quad f(1,0,1) = (0,0), \quad f(0,1,1) = (2,1),$ determine $f(x,y,z)$ for all $(x,y,z) \in \mathbb{R}^3$.

(2×15=30)