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Reg. No.................................

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M.Sc. DEGREE (C.S.S.) EXAMINATION, DECEMBER 2018

First Semester

Faculty of Science

Branch I (a) : Mathematics

MT01C01—LINEAR ALGEBRA

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries weight 1.

1. Are the vectors (1, 8, 2, 3), (– 2, – 1, 5, 2), (3, – 1, – 4, 1), (2, 0, 1, 3) linearly independent in $\mathbb{R}^4$.
2. Find three vectors in $\mathbb{R}^3$ which are linearly independent, and none of them lies on the axes.
3. Let V be the vector space of all $n \times n$ matrices over the field F, and let B be a fixed $n \times n$ matrix. Is T : V $\to$ V defined by T (A) = AB – BA, a linear map ?
4. Consider the linear operator T defined by $T(x_1, x_2, x_3) = (3x_1, x_1 - x_2, 2x_1 + x_2 + x_3)$. Prove that $(T^2 - I)(T - 3I) = 0$.
5. Show that linear combination of n-linear functions is n-linear.
6. Check whether the function D on the set of 3 × 3 matrices over the field of real numbers defined by $D(A) = A_{11} A_{22} A_{33}$ is 3-linear.
7. Find a projection E which projects $\mathbb{R}^2$ onto the subspace spanned by (1, 1) along the subspace spanned by (1, – 2).
8. Find a 3 × 3 matrix for which the minimal polynomial is x^2.

(5 × 1 = 5)

Turn over

Part B

Answer any **five** *questions.*
Each question carries weight 2.

9. Let $\mathscr{B} = \{(1, 0, -1), (1, 1, 1), (1, 0, 0)\}$ be the ordered basis for $\mathbb{R}^3$. Find the co-ordinates of the vector (a, b, c) with respect to the basis $\mathscr{B}$.

10. Let W_1 and W_2 are subspaces of a finite-dimensional vector space V. Prove that $(W_1 \cap W_2)^0 = W_1^0 + W_2^0$.

11. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3$. Find the matrix of T in the standard basis.

12. Suppose V and W are finite dimensional vector spaces over the field F, and $T : V \to W$ be a linear transformation. Show that the range of T^t is the annihilator of the null space of T.

13. Let K be a commutative ring with identity, and let A and B be $n \times n$ matrices over K. Prove that

$$\det(AB) = (\det A)(\det B).$$

14. If E is a projection on R along N, then prove that (I – E) is a projection on N along R.

15. Let a, b and c be elements of a Field F and $A = \begin{bmatrix} 0 & 0 & c \\ 1 & 0 & b \\ 0 & 1 & a \end{bmatrix}$ be the matrix over F. Prove that the characteristic polynomial and minimal polynomial for A is $x^3 - ax^2 - bx - c$.

16. Let W be an invariant subspace for T. Prove that the characteristic polynomial of the restriction operator T_W divides the characteristic polynomial of T.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question carries weight 5.

17. Let V be the set of all 2×2 matrices A with complex entries which satisfy $A_{11} + A_{22} = 0$. Show that V is a vector space over the field of real numbers, with the usual matrix addition and multiplication of a matrix by a scalar. Also find a basis for this vector space.

18. If f is a non-zero linear functional on the vector space V then, show that the null space of f is a hyperspace in V.

19. Let T be the linear operator on $\mathbb{R}$, the matrix of which in the standard ordered basis is $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$.

    Find a basis for the range of T and a basis for the null space of T.

20. Let K be a commutative ring with identity and let n be a positive integer. Show that there is precisely one determinant function on the set of $n \times n$ matrices over K.

21. Explain the Direct-sum decomposition of a finite dimensional vector space.

22. Let V be a real vector space and E a projection. Prove that (I + E) is invertible. Find its inverse.

(3 × 5 = 15)