QP CODE: 19002356

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Reg No : ....

Name : ....

M.Sc. DEGREE (C.S.S) EXAMINATION, NOVEMBER 2019

First Semester

Faculty of Science

MATHEMATICS

Core - ME010105 - GRAPH THEORY

2019 Admission Onwards

EC02C51B

Time: 3 Hours

Maximum Weight :30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define (a) complete bipartite graph (b) selfcomplementary graph (c) clique of a graph (d) isomorphism between graphs
2. Define orientation of a graph and how many orientations does a simple graph of m edges have?
3.
   a. Define connectivity and edge connectivity of a graph.
   b. Prove or disprove: if H is a subgraph of G (i) $\kappa(H) \leq \kappa(G)$

      (ii) $\lambda(H) \leq \lambda(G)$
4. a.Define and give example for cyclic edge connectivity of a graph
   b.State Ear decomposition theorem of a block.
5. Prove that a simple graph G is a tree if and only if any two distinct vertices are connected by a unique path.
6. Define Eulerian graph with an example.
7. If G contains exactly one odd cycle, then show that $\chi(G) = 3$.
8. Prove that every k chromatic graph contains a k critical subgraph.
9. *Draw dual of* W_5 *and write your comment.*
10. *What is the spectrum of* K_n

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Define graphical sequence and write the necessary condition for $d = \{d_1, d_2, \ldots, d_n\}$ to be graphical. Show that $d = ($ $\{7, 6, 3, 3, 2, 1, 1, 1\}$) is not graphical.

12. For a simple graph G prove that $m(L(G)) = \frac{1}{2}\sum_{i=1}^{n} d_i^2 - m$

13. If e is a loop of a connected graph G, then prove $\tau(G) = \tau(G-e) + \tau(Goe)$

14. Find a minimal spanning tree of G whose weight matrix is given by

$$\begin{bmatrix} \infty & 385 & 425 & 1035 & 708 & 644 \\ 385 & \infty & 255 & 740 & 773 & 329 \\ 425 & 255 & \infty & 679 & 531 & \infty \\ 1035 & 740 & 679 & \infty & 816 & 860 \\ 708 & 773 & 531 & 816 & \infty & 1095 \\ 644 & 329 & \infty & 860 & 1095 & \infty \end{bmatrix}$$ using Kruskal's algorithm

15. Let G be a simple graph with $n \geq 3$ vertices. If for every pair of nonadjacent vertices u,v of G, $d(u) + d(v) \geq n$, then show that G is Hamiltonian.

16. For any graph G with n vertices and independence number α, prove that $n/\alpha \leq \chi \leq n - \alpha + 1$.

17. *Find a simple graph G with degree sequence (4, 4, 3, 3, 3, 3) such that (i) G is planar (ii) G is nonplanar.*

18. *Prove that for any simple planar graph G,* $\delta(G) \leq 5$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) Define strong product of two graphs G_1 and G_2.
b) Find the order and size of $G_1 \boxtimes G_2$
c) Construct $K_2 \boxtimes P_3$

20.
a. If {x,y} is a 2-edge cut of a graph G, show that every cycle of G that contains x must also contain y.
b. Simple connected cubic graph G has a cut vertex if and only if it has a cut edge.
c. Show that a graph has a cut vertex need not imply it has a cut edge

21. (a) If G is a simple graph with $n \geq 3$ vertices such that $d(u) + d(v) \geq n + 1$ for every pair of non adjacent vertices u and v of G, then G is hamiltonian connected. Prove

(b) Show by an example that if closure of a graph G is complete then G is Hamiltonian.

(c) Show by an example that if closure of a graph G is Hamiltonian then G is Hamiltonian.

22. What you mean by four color conjecture. Prove that every planar graph is 6 – vertex colorable.

(2×5=10 weightage)