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Reg. No..............................

Name..................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, DECEMBER 2018

First Semester

Faculty of Science

Branch I (a)—Mathematics

MT 01 C04—GRAPH THEORY

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries weight 1.

1. Give an example of a non-simple disconnected graph with $\delta \geq \frac{n-1}{2}$.
2. Describe Edge connectivity with examples.
3. Give an example of a tree with just one central vertex that is also a centroidal vertex.
4. Describe a minimal dominating set of the Petersen graph.
5. Define Chromatic number of a graph G and show that χ (G) = 2 if and only if G is a bipartite graph with at least one edge.
6. Does there exist an Eulerian graph with an even number of vertices and an odd number of edges ? Justify.
7. Is the Petersen graph planar ? Justify.
8. Draw the dual of the Herschel graph.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question carries weight 2.

9. Let G be a simple connected graph with n vertices such that *Aut* (G) $\simeq$ S_n, show that G is the complete graph K_n.
10. In a group of six people, prove that there must be three people who are mutually acquainted or three people who are mutually non-acquainted.
11. Suppose G is a connected graph of order $n \geq 2$, prove that $\gamma(G) \leq \frac{n}{2}$.

Turn over

12. Prove that a simple graph is a tree if and only if any *two* distinct vertices are connected by a unique path.
13. Prove that any critical graph is connected.
14. For a non-trivial connected graph G, show that G is Eulerian if and only if G is an edge-disjoint union of cycles.
15. Prove that a graph is planar if and only if it is embeddable on a sphere.
16. Show that all wheels W_n, $n \geq 3$ are self-dual.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries weight 5.

17. Prove that the set *Aut* (G) of all automorphisms of a simple graph G is a group with respect to the composition of mappings as the group operation.
18. Prove that every connected graph contains a spanning tree.
19. Show that a tree with at least two vertices contains at least two pendant vertices.
20. Prove that a graph G is Eulerian if and only if each edge e of G belongs to an odd number of cycles of G.
21. Prove that a graph G is planar if and only if each of its blocks is planar.
22. Prove that K_5 is non-planar.

(3 × 5 = 15)