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Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, FEBRUARY 2021

Third Semester

Faculty of Science

Branch I (A)–Mathematics

MT 03 C12—FUNCTIONAL ANALYSIS

(2012 – 2018 Admissions)

Time : Three Hours — Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Prove *or* disprove : Union of two subspace of a vector space is again a subspace.
2. Show that in a Banach space an absolutely convergent series is convergent.
3. Differentiate between linear functional and linear operator with examples.
4. With usual notation prove :

$$\| x + y \|^2 \leq \| x \|^2 + \| y \|^2 .$$

5. Show that every orthonormal set is linearly independent.
6. Define Banach space and Hilbert space. Give an example of a Banach space which is not Hilbert.
7. State category theorem. Explain its importance.
8. When a normed space become complete ? Prove your statement.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Show that every finite dimensional subspace of a normed space is complete.
10. Show that if a normed space has a Schauser basis, it is separable.
11. State and prove the theorem on the bounded linear extension of bounded linear operator.
12. Establish the theorem on the dimension of the algebraic dual of a n-dimensional vector space.

Turn over

13. Obtain Schwarz inequality. When it becomes an equality ?

14. Define : (i) lattice ; (ii) partially ordered set ; (iii) totally ordered set ; (iv) convex set.

15. Show that every Hilbert space is reflexive.

16. State and prove uniform boundedness theorem.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Let $T : X \to Y, S : Y \to Z$ be bijective linear operators. Show that :

    (a) $(S\,T)^{-1} = T^{-1}S^{-1}$.

    (b) The range $\mathscr{R}$ (T) is a vector space.

    (c) If dim $\mathscr{D}$ (T) = $n < \infty$, then dim $\mathscr{R}$ (T) = dim $\mathscr{D}$ (T).

18. Let X and Y be two normed space. Define B (X, Y) and show that it is a Banach space if, Y is Banach. Deduce that the dual X^1 of the normed space X is a Banach space.

19. (a) Obtain the dual space of l^p where $1 < p < \infty$.

    (b) Show that inner product spaces are normed spaces but not conversely.

20. (a) Establish the existence of Hilbert-space adjoint operator.

    (b) With usual notations prove $(S + T)^* = S^* + T^*$ and $(S\,T)^* = T^*\,S^*$.

21. (a) Obtain Bessel inequality.

    (b) Define unitary operators. State and prove four of its properties.

22. Establish Hahn Banach theorem (Generalized).

$(3 \times 5 = 15)$