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Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2019

Third Semester

Faculty of Science

Branch I (A)—Mathematics

MT 03 C 13—DIFFERENTIAL GEOMETRY

(2012–2018 Admissions)

Time : Three Hours

Maximum Weight : 30

Part A

*Answer any five questions.
Each question has weight 1.*

1. How will you visualize the graph of a function $f : U \rightarrow \mathbb{R}$, $U \subset \mathbb{R}^2$, given its level sets.
2. Give an example to show that the set of vectors tangent at a point p of a level set might be all of \mathbb{R}_p^{n+1} .
3. Define Gauss map and sketch it for 1 – surface of \mathbb{R}^2 .
4. Prove that in an n -plane, parallel transport is path independent.
5. Find the curvature of $\vec{r} = (\cos 2\mu, \sin 2\mu, 2 \sin \mu)$.
6. Explain normal curvature and oriental n -surface.
7. Explain global property with example.
8. Explain : Curvature of surfaces.

(5 × 1 = 5)

Part B

*Answer any five questions.
Each question has weight 2.*

9. Show that a connected n -surface SCR^{n+1} has exactly two smooth unit normal vector fields.
10. State and prove the theorem on the existence and uniqueness of integral curves. Extend it to n -surface.
11. Show that a geodesic can be found to pass through any given point and have a given direction on a surface. Show also that the geodesic is uniquely determined by the initial conditions.
12. Show that parallel transport from p to q along a piecewise smooth parametrized curve is a vector space isomorphism which preserves dot product.

Turn over





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13. Find the arc length of one complete turn of the circular helix

$$r(u) = (a \cos u, a \sin u, b u) - \infty < u < \infty.$$

14. Establish necessary and sufficient condition for a global parametrization of an oriented plane curve.
15. Show that parametric equations of a surface need not be unique.

16. Let S be the ellipsoid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$, $(a, b, c \neq 0)$, oriented by its outward normal. Find the Gaussian curvature.

(5 × 2 = 10)

Part C

*Answer any **three** questions.
Each question has weight 5.*

17. Define covariant derivative of a vector \bar{X} . Establish necessary and sufficient conditions for a parametrised curve $\alpha : I \rightarrow S$ be a geodesic.
18. (a) Show that the set $SL(3)$ of all 3×3 real matrices with determinant equal to 1 is an 8-surface in \mathbb{R}^9 .

(b) What is the tangent space to $SL(3)$ at $p = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$?

19. Define Weingarten map and prove it is self adjoint.
20. Let C be a connected oriented plane curve and $\beta : I \rightarrow C$ be a unit speed global parametrisation of C . Prove that β is 1-1 or periodic.
21. (a) State and prove inverse function theorem on n -surfaces.
(b) Differentiate between curvature of plane curves and curvature of surfaces.
22. Establish the two theorems to show that surfaces and parametrised surfaces are the same.

(3 × 5 = 15)

