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Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, FEBRUARY 2021

Third Semester

Faculty of Science

Branch I (A)–Mathematics

MT 03 C13—DIFFERENTIAL GEOMETRY

(2012–2018 Admissions)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Find the level sets for $f(x_1, x_2, \ldots x_n) = x_1^2 + x_2^2 + \ldots + x_n^2$.
2. What is the tangent space to SL(3) at $p = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.
3. Explain a geodesic in an n surface SCR^{n+1}. Also explain a geodesic flow.
4. Show that in an n-plane, parallel transport is path independent.
5. Compute the Weingarten map for the circular cylinder $x^3 + y^3 = a$ in R^* where $a \neq 0$.
6. Explain line integral with example.
7. Define convex surface and parametrized surface with example.
8. Explain global property with an example.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Find the integral curve through $p = (1, 1)$ of the vector field $\mathrm{X}(x_1, x_2) = \left(-2x_2, \frac{1}{2}x_1\right)$.
10. Define the graph of a function. Show that the graph of any function $f : \mathrm{R}^n \to \mathrm{R}$ is a level set for some function $\mathrm{F} : \mathrm{R}^{n+1} \to \mathrm{R}$.

Turn over

11. Prove that the velocity vector field along a parametrised curve α is paralled iff α is a geodesic.

12. Find the velocity speed and acceleration of the parametrized curve $\alpha(t) = (\cos t, \sin t, t_2)$.

13. Evaluate L_p for $x_2^2 + x_3^2 = a^2$ in R^3.

14. Compute $\nabla_v f$ where $f(x_1, x_2) = 2x_1^2 + 3x_2^2$ and $v = (1,0,2,1)$.

15. State and prove inverse function theorem for n-surface.

16. Show that parametric equation of a surface are not unique.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) Show that the unit n-sphere $x_1^2 + x_2^2 + \ldots x_{n+1}^2 = 1$ is a connector if $n > 1$.

    (b) Explain (i) integral curve ; (ii) n-surface ; (iii) vector field ; (iv) orientation.

18. (a) Show that parallel transport from p to q along a piecewise smooth parametrised curve is a vector space isomorphism which preserves dot products.

    (b) Explain covariant derivative with example.

19. Prove that the Weingarten map is self-adjoint.

20. Define arc length with an example. Also establish the existence of arc length.

21. Obtain necessary and sufficient conditions for $f : S \to R^K$ to be smooth where $S \subset R^{n+1}$ is n-surface.

22. Establish the local equivalence of surfaces and parametrised surfaces. Illustrate with example.

(3 × 5 = 15)