QP CODE: 20100569 20100569 **Reg No** : ....

Name : ....

BSc DEGREE (CBCS) EXAMINATION, MARCH 2020

Sixth Semester

Core course - MM6CRT03 - COMPLEX ANALYSIS

B.Sc Mathematics Model I,B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

69EDE29F

Time: 3 Hours Maximum Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the imaginary part of the function f(z)=$\tan z$
2. Show that f(z) is continuous at $z_{0,}$ so is |f(z)|
3. Solve the equation e^z=1+i
4. Find i^i and its principal value
5. Evaluate $\cosh^{-1}$(-1)
6. Define Simple closed curve.
7. If C is any simple closed contour, then evaluate $\int_C exp(z^3)dz$.
8. Evaluate $\displaystyle\int_{|z|<2} \frac{ze^z}{(z^2+9)^5}dz$
9. Evaluate $lim_{n\to\infty} z_n$ where$z_n = \frac{-2+i(-1)^n}{n^2}$
10. Find the Laurent's series that represents the function $f(z) = z^2 sin(\frac{1}{z^2})$ in the domain $0 <| z |< \infty$, given the expansion of $\sin z$
11. Define isolated singular points of a complex function with an example.
12. Show that the existence of Cauchy Principal Value does not imply the existence of $\int_{\infty}^{-\infty} f(x)dx$

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove that f(z)=$f(z) = \begin{cases} \frac{Im\, z^2}{|z|^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$ satisfies CR equations and is not differentiable at z=0

14. Find the harmonic conjugate of v=log(x^2+y^2)+x-2y

15. Expand sin z using exponentials

16. State and prove Cauchy's integral formula.

17. If f(z) is analytic within and on a circle C given by $|z - z_0| = R$ and if $|f(z)| \leq M$ for every z on C,Prove that $|f^n(z_0)| \leq M\frac{n!}{R^n}$

18. State and prove maximum modulus principle.

19. *Obtain a power series expansion of* e^z *in powers of* $z - 1$ *when* $|z-1| < \infty$.

20. Define the three types of isolated singularities of a complex function $f(z)$.

21. Find the residue at $z = 0$ of $of\, f(z) = \frac{1}{z(e^z-1)}$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a) If f(z)=u(x,y)+iv(x,y) be an analytic function.Then prove the following:
    i) u(x,y)=constant implies f(z) is constant
    ii) v(x,y)=constant imples f(z) is constant
    iii) |f(z)|=constant implies f(z) is constant
    iv) arg(f(z))=constant implies f(z) is constant
    b) Find an analytic function f(z) with real part x^3-3xy^2

23. Evaluate $\int_c f(z)dz, \text{where } f(z) = \exp(\pi\bar{z})$ and C is the boundary of the square with vertices at the points 0, 1, 1+i and i, the orientation of C being in the counter clockwise direction.

24. *(a) Assuming the series expansion of* e^z, *find a Maclaurin series expansion of* $\sin z$
    (b) Use the series expansion of $\sin z$ *to obtain the series expansions of* $\cosh z$ *and* $\sinh z$ *about* $z_0 = 0$
    (c) Hence deduce an expansion of $\cosh z$ *about* $z_0 = -2\pi i$

25. State and prove Cauchy's Residue Theorem. Using the theorem, evaluate $\int_C \frac{e^{-z}}{z^2} dz$, where C is the circle $|z| = 3$

(2×15=30)