QP CODE: 22000353

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Reg No : ....

Name : ....

MSc DEGREE (CSS) EXAMINATION , JANUARY 2022

Second Semester

CORE - ME010204 - COMPLEX ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

EEFB149C

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. State Abel's convergence theorem.
2. Define symmetry wth respect to a circle.
3. Evaluate $\int_{|z|=r} x dz$ for the positive sense of the circle.
4. Define a rectifiable arc.Also state the necessary and sufficient condition for an arc to be rectifiable.
5. State Cauchy's theorem for a rectangle with exceptional points.
6. State the Cauchy's integral formula for higher derivatives. Evaluate $\int\limits_{|z|=2} \frac{z^2}{(z+1)^3} dz$.
7. Define removable singularity of a function. Give an example.
8. State the maximum principle.
9. Define a multiply connected region.
10. Define residue of $f(z)$ at an isolated singularity.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Obtain the complex form of Cauchy –Riemann equations.
12. If z_1, z_2, z_3, z_4 are distinct points in the extended plane and T any linear transformation, then prove that $(Tz_1, Tz_2, Tz_3, Tz_4) = (z_1, z_2, z_3, z_4)$.

13. If f(z) is analytic in an open disk Δ, then prove that $\int_\gamma f(z)dz = 0$, for every closed curve γ in Δ.

14. State and prove Cauchy's integral formula.

15. Prove that a nonconstant analytic function has no zeros of infinite order.

16. If $f(z)$ is analytic with $f'(z_0) \neq 0$, prove that it maps a neighborhood of z_0 conformally and topologically onto a region.

17. If $f(z)$ is analytic and non zero in a simply connected region Ω , then it is possible to define a single valued analytic branch of $log f(z)$ and $\sqrt[n]{f(z)}$ in Ω.

18. State and prove generalized argument principle.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (i) Show that z and z' correspond to diametrically opposite points on the Riemann sphere if and only if $z\overline{z}' = -1$.

(ii) Find the correspondence between the coordinates of a point on the Riemann sphere and its image in the complex plane.

20. i) If the piecewise differentiable closed curve γ does not pass through the point 'a' ,then prove that the valueof the integral $\int_\gamma \frac{dz}{z-a}$ is a multiple of 2πi.
ii) If γ is a closed curve, then prove that index n(γ,a) is a constant in each of the regions determined by γ.

21. (a) State and prove the Weirstrass's theorem for essential singularities.
(b) Show that the function which is analytic in the whole plane and has a non essential singularity at $z = \infty$ reduces to a polynomial.

22. Evaluate $\int_0^\infty \frac{dx}{x^4+a^4}, a > 0.$

(2×5=10 weightage)