QP CODE: 21100560

21100560

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS)EXAMINATION, MARCH 2021

Third Semester

Core Course - MM3CRT01 - CALCULUS

Common to B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

AA22FEDD

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. State Taylor's Therom.
2. Define point of inflection.
3. Find $\frac{ds}{dx}$ for the curve $y = \cosh(\frac{x}{c})$.
4. Find the asymptotes parallel to co-ordinate axes of the curve $x^4 + x^2 y^2 - a^2 (x^2 + y^2) = 0$
5. Find $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ if $f(x, y) = x^2 - y^2$
6. If $w = f(x)$ and $x = g(r, s)$, then what will be $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$
7. Explain the method of Lagrange multipliers to find the extreme values of a function $f(x, y, z)$ subject to a constraint $g(x, y, z) = 0$
8. If $R(y)$ and $r(y)$ denote the outer and inner radius of cross section of a solid of revolution about Y-axis, with hole at y ; $c \leq x \leq d$. Find the volume of solid.
9. Explain Shell formula for finding volume of solid obtained by revolving a bounded region about a vertical line .
10. The line segment $x = 1 - y;\ 0 \leq y \leq 1$ is revolved about the Y-axis to generate the cone. Find its lateral surface area (which excludes base area).
11. State Fubini’s theorem (First form).
12. Find the cylindrical coordinate equation for the cylinder $x^2 + (y - 1)^2 = 1$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Expand $\sin^{-1} x$ using Maclaurin's series.

14. Find the envelope of family of straight line y = mx + $\sqrt{a^2m^2+b^2}$,m being the parameter.

15. Find all local extreme values and saddle point, if any, of the function $f(x,y)=x^2+y^2-xy-2x.$

16. Find the absolute maximum and minimum values $f(x,y)=x^2+y^2$ on the triangular plate in the first quadrant bounded by the lines $x=0,\ y=0,\ y+2x=2.$

17. A pyramid $\sqrt{3}$ m high has a square base that is $\sqrt{3}$ m on a side. The cross section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.

18. The region between the curve $y=\sqrt{x}$; $0\le x\le 4$ and the X-axis is revolved about the X-axis to generate a solid . Find its volume.

19. Sketch the region of integration and write an equivalent double integral of $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3\,y\,dx\,dy$ with the order of integration reversed.

20. Find the area of the region R bounded by $y=2x^2$ and $y^2=4x$.

21. Find the average value $f(x,y,z)=x^2+9$ over the cubical region D bounded by the coordinate planes $x=2, y=2$ and $z=2$ in the first octant.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a) Find the co-ordinates of the centure of curvature at the point (x,y) on the parabola $y^2=4ax$ and hence find its evolute.

    b) In the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, show that the radius of curvature at an end of the major axis is equal to semi-latus rectumof the ellipse.

23. (a). If $u=\sin^{-1}\left(\dfrac{x-y}{x+y}\right)$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=0$

(b). Verify that $\dfrac{\partial^2 w}{\partial x \partial y} = \dfrac{\partial^2 w}{\partial y \partial x}$ if $w = x^y + \sin(xy)$

(c). Find the point $P(x, y, z)$ closest to the origin on the plane $2x + y - z = 5$

24. (a). The region bounded by the curve $y = \sqrt{4x - x^2}$, the X-axis and the line $x = 2$ is revolved about the X-axis to generate a solid. Find its volume.

(b). Find the length of the arc of the semi cubical parabola $y^2 = x^3$ extending from the origin to the point $(1, 1)$.

25. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x+y}\,(y - 2x)^2 dy\ dx$ by applying the transformation $u = x + y$, $v = y - 2x$.

(2×15=30)