# B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, APRIL 2022 Third Semester 

## Core Course - MM3CRT01-CALCULUS

Common to B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science 2017 Admission Onwards

805BD574
Time: 3 Hours
Max. Marks : 80

## Part A

Answer any ten questions.
Each question carries 2 marks.

1. Expand $e^{x}$ using Maclaurin's series.
2. Find the points of inflection of the curve $y=x^{3}-9 x^{2}+7 x-6$.
3. Find the radius of curvature at any point on the curve $s=c \tan \psi$
4. Define asymptotes of a curve.
5. If $f(x, y)=x-y$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$
6. State the first derivative test for local extreme values.
7. Write down the Lagrange multipliers equation to find the extreme values of a function $f(x, y, z)$ subject to two constraint $g_{1}(x, y, z)=0$ and $g_{2}(x, y, z)=0$
8. Evaluate the volume of the solid of cross sectional area $A(x)=2+x^{2}$ from $x=1$ to $x=2$.
9. Write down the formula for calculating the volume of solid of revolution about the X -axis and Y -axis.
10. Find the length of the curve $y=x \sqrt{3}+1$ from $x=1$ to $x=2$.
11. Sketch the region of integration and evaluate the integral

$$
\int_{\pi}^{2 \pi} \int_{0}^{\pi}(\sin x+\cos y) d x d y
$$

12. Define the Jacobian $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.
$(10 \times 2=20)$

## Part B

Answer any six questions.
Each question carries 5 marks.
13. Find the Taylor series generated by $f(x)=1 / x$ at $x=2$. Discuss the convergence of the series.
14. Find the equation of the circle of curvature at the point $(0, b)$ of the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$.
15. Show that the function satisfies Laplace equation $f(x, y)=e^{x}(x \cos y-y \sin y)$
16. Evaluate $\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y} \frac{\partial w}{\partial z}$ in terms of $x, y, z$ if
$w=\frac{p-q}{q-r}, p=x+y+z, q=x-y+z, r=x+y-z$,
17. Find the volume of the solid generated when the region under the curve $y=x^{2}$ over the interval $[0,2]$ is rotated about the line $y=-1$ using Washer method..
18. Find the area of the surface generated by revolving the curve
$x=y^{2} ; 1 \leq x \leq 9$, about the X -axis.
19. Sketch the region of integration and write an equivalent double integral of $\int_{0}^{2} \int_{-\sqrt{4-x^{2}}}^{\sqrt{4-x^{2}}} 5 x d y d x$ with the order of integration reversed.
20. Find the volume of the region cut from the cylinder $x^{2}+y^{2}=4$ by the planes $z=0$ and $x+z=3$.
21.

Evaluate the cylindrical coordinate integral $\int_{0}^{2 \pi} \int_{0}^{\frac{\theta}{2 \pi}} \int_{0}^{3+24 r^{2}} d z r d r d \theta$

## Part C

Answer any two questions.
Each question carries 15 marks.
22. (a). Find the evolute of the rectangular hyperbola $x y=1$
(b). If $(X, Y)$ be the coordinates of centre of curvature of the curve
$\sqrt{x}+\sqrt{y}=1$ at $(a, b)$, then prove that $X+Y=3(a+b)$.
23. (a). Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x, y)=x \tan ^{-1}(x y)$
(b). Find the shortest distance from the origin to the hyperbola $x^{2}+8 x y+7 y^{2}=225$.
24. Compute the volumes of the solid generated by revolving the region bounded by $y=x$ and $y=x^{2}$ about each coordinate axis using (i) the shell method (ii) the washer method.
25. (a). Evaluate the Jacobian of the transformation from cylindrical coordinate system $(r, \theta, z)$ to rectangular system $(x, y, z)$.
(b). Evaluate $\iint_{R} x y\left(x^{2}+y^{2}\right)^{3 / 2} d A$ where R is the region in the first quadrant bounded by the circle $x^{2}+y^{2}=1$.

