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Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, APRIL 2019

Fourth Semester

Faculty of Science

Branch I (a)–Mathematics

MT0 4E 02—COMBINATORICS

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions from this part.*
Each question has weight 1.

1. State principle of addition and multiplication.
2. Find the number of ways to seat n married couples around a table in each of the following cases :
   (a) Men and women alternate.
   (b) Every women is next to her husband.
3. Show that among any group of 13 people, there must be atleast 2 whose birthdays are in the same month.
4. Show that $R(3,4) \leq 9$.
5. Show that $|A \cup B \cup C| = (|A| + |B| + |C|) - (|A \cap B| + |A \cap C| + |B \cap C|) + |A \cap B \cap C|$.
6. Find the number of integers in the set $\{1, 2, \ldots 10^3\}$ which are not divisible by 5 nor by 7 but are divisible by 3.
7. Show that the exponential generating function for the sequence $(1, 1\cdot 3, 1\cdot 3, 1\cdot 3\cdot 5, 1\cdot 3\cdot 5\cdot 7, \ldots\ldots)$ is $(1-2x)^{-3/2}$.
8. Find the coefficient of x^{20} in the expansion of $\left(x^3 + x^4 + x^5 + \ldots\right)^3$.

(5 × 1 = 5)

Turn over

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Between 20,000 and 70,000, find the number of even integers in which no digit is repeated.

10. Show that $S(r,n) = S(r-1,n-1) + nS(r-1,n)$ where $r,n \in N$ with $r \geq n$.

11. Define Ramsey numbers. Also show that :

    (i) $R(p,q) = R(q,p)$.

    (ii) $R(1,q) = 1$.

    (iii) $R(2,q) = q$.

12. Let $A = \{a_1, a_2, \ldots a_5\}$ be a set of 5 positive integers. Show that for any permutation $a_{i_1}\ a_{i_2}\ a_{i_3}\ a_{i_4}\ a_{i_5}$ of A, the product $(a_{i_1} - a_1)(a_{i_2} - a_2)\ldots(a_{i_5} - a_5)$ is always even.

13. Three identical black balls, four identical red balls and 5 identical white balls are to be arranged in a row. Find the number of ways that this can be done if all the balls with the same colour do not form a single block.

14. Find the number of integer solutions to the equation $x_1 + x_2 + x_3 = 28, 3 \leq x_1 \leq 9, 0 \leq x_2 \leq 8, 7 \leq x_3 \leq 17$.

15. Solve $a_n - 4a_{n-1} + 4a_{n-2} = 0$, given that $a_0 = \frac{-1}{4}, a_1 = 1$.

16. Find the number of ways to distribute 40 identical balls to 7 distinct boxes if box 1 must hold atleast 3, and atmost 10 of the balls.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) Consider a collection of r objects in which r_1 are of type I, r_2 are of type 2 . . . and r_n are of type n, where $r_1 + r_2 + \ldots + r_n = v$. Show that $P(r, r_1, r_2, \ldots r_n) = \frac{r!}{r_1! r_2! \ldots r_n!}$.

    (b) Find the number of ways to pure a 1 × 7 rectangle by 1 × 1, 1 × 2 and 1 × 3 blocks, assuming that blocks of the same size are indistinguishable.

18. Let ABC be an equilateral triangle and ε the set of all points contained in the three segments AB, BC, CA (including A, B and C). Show that for every partition of ε into 2 disjoint subsets, at least one of the 2 subsets contains the vertices of a right-angled triangle.

19. In how many ways can a committee of 5 be formed from a group of 11 people consisting of 4 teachers and 7 students if :

   (i) There is no restriction in the selection.

   (ii) The committee must include exactly 2 teachers.

   (iii) The committee must include at least 3 teachers.

   (iv) A particular teacher and a particular student cannot be both in the committee.

20. (a) Let S be an n-element set and let $\{P_1, P_2, \ldots P_q\}$ be a set of q properties for elements of S. Show that for each $m = 0, 1, 2 \ldots, q$:

$$E(m) = w(m) - \binom{m+1}{m} w(m+1) + \binom{m+2}{m} w(m+2) - \ldots + \ldots + (-1)^{q-m} \binom{q}{m} w(q).$$

(b) Let $A_1, A_2 \ldots A_q$ be any q subsets of a finite set S, show that :

$$\left| \bar{A}_1 \cap \bar{A}_2 \ldots \cap \bar{A}_q \right| = |S| - \sum_{i=i}^{q} |A_i| + \sum_{i<j} \left| A_i \cap A_j \right| \ldots + (-1)^q \left| A_1 \cap A_2 \ldots \cap Aq \right|$$ where $\bar{A}_i$ denotes the complement of A_i in S.

21. Solve the recurrence relation $a_n - 3a_{n-1} = 2 - 2n^2$, given $a_0 = 3$.

22. If $n = P_1^{m_1} P_2^{m_2} \ldots P_k^{m_n}$ be a factorization of n, prove that $\phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \ldots \left(1 - \frac{1}{p_n}\right)$.

(3 × 5 = 15)