QP CODE: 21002035

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2021

First Semester

Faculty of Science

CORE - ME010103 - BASIC TOPOLOGY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

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Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define a metric space on a non empty set. Give an example of a metric which is translation invariant
2. Define order topology
3. Define Product topology in finite case
4. Define Closed set, Clopen set and Dense set of a topological space.
5. Prove that $\bar{A} = A \cup \partial(A)$,where $\partial(A)$ denotes boundary of A.
6. *Show that every open surjective map is a quotient map.*
7. Define separable space. Give an example.
8. Let X be a compact topological space and $f : X \to Y$ be continuous and onto. Prove that Y is compact.
9. Define a path in topological space and hence define a path connected space. Give an example of a space which is connected but not path connected
10. *Write down the equivalent conditions of normality*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Prove that convergence of sequence is a co countable topology behaves the same as the way in a discrete space
12. Characterize base of a topological space

13. Let the function $f: X \to Y$ is $\mathfrak{I} - \mathcal{U}$ continuous, where $\mathfrak{I}, \mathcal{U}$ be topologies on X, Y respectively. Then show that for any closed subset A of Y, $f^{-1}(A)$ is closed in X *if and only if* for all A ⊆ X, $f(\bar{A}) \subseteq \overline{f(A)}$.

14. . Let $f: X \to Y$ be a function, where X, Y are topological spaces. Prove that (i) f is continuous if and only if f^{-1} is open (ii) f^{-1} is continuous if and only if f is open.

15. Given that a topological space X cannot be written as disjoint union of two non - empty opensets. Prove that X is connected.

16. Let X_1, X_2 be connected topological spaces. Prove that $X_1 \times X_2$ is connected.

17. *Show that X is a T_1 space if and only if the topology on X is stronger than the co finite topology on X*

18. *Show that limit of a sequence in a Hausdorff space is unique. What is the converse of the statement.? Justify your answer*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Show that metrisability is a hereditary property

20. *(i) Define Weak topology*
(ii) Define Strong topology
(iii) Show that product topology is the weak topology determined by the projection functions.

21. *Let X,Y be topological spaces $x \in X$ and f: X →Y a function. Suppose X is first countable at x. Prove that f is continuous at x if and only if for every sequence $\{x_n\}$ which converges to x in X, the sequence $\{f(x_n)\}$ converges to $f(x)$ in Y .*

22. Show that quotient space of a locally connected space is locally connected

(2×5=10 weightage)