18002222

18002222

Reg. No................................

Name...................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, DECEMBER 2018

First Semester

Faculty of Science

Branch I (a) : Mathematics

MT01C02—BASIC TOPOLOGY

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries weight 1.

1. Prove that a second countable space always contains a dense subset.
2. Prove that a set is closed if and only if it contains its boundary.
3. Define quotient map. Show that every quotient space of a discrete space is discrete.
4. Show that every continuous image of a compact space is compact.
5. If X is a locally connected topological space, show that components of open subsets of X are open in X.
6. Show that closure of a connected space is connected.
7. Show that complete regularity property is hereditary.
8. Show that every compact Hausdorff space is τ_4.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question carries weight 2.

9. Prove that open sphere in a metric space X is an open set.
10. For a set A in a topological space X, prove that $\bar{A}$ is the disjoint union of int (A) with the boundary of A
11. Prove that composition of two quotient maps is again a quotient map.
12. Show that every continuous real valued function on a compact space is bounded and it attains its extrema.
13. Prove that every quotient space of a locally connected space is locally connected.

Turn over

14. Show that any subset of $\mathbb{R}$ is connected if and only if it is an interval.

15. Suppose X be a completely regular space, and F (compact), C (closed) subsets of X such that $F \cap C = \emptyset$. Prove that there exist a continuous function from X into the unit interval which takes the value 0 at all points in F and the value 1 at all points of C.

16. Prove that every Tychonoff space if τ_3.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries weight 5.

17. (a) Show that for a given family **S** of subsets of X, there is a unique topology τ on X having **S** as the sub-base.

    (b) Suppose τ_1, τ_2 are topologies on X and $Y \subset X$ respectively. Show that if τ_1 is stronger than τ_2 then τ_1/Y is stronger than τ_2/Y.

18. (a) Show that every second countable space is first countable.

    (b) Let X, Y be topological spaces, $x \in X$ and $f: X \to Y$ a function. Suppose X is first countable at x. Show that f is continuous at x if and only if for every sequence $\{x_n\}$ which converges to $x \in X$, the sequence $\{f(x_n)\}$ converges to $f(x) \in Y$.

19. (a) Prove that the topological product of a finite number of pathconnected spaces is pathconnected.

    (b) Show that every non-empty proper subset of X has a non-empty boundary if and only if X is connected.

20. (a) Show that every space is the disjoint union of its components.

    (b) Prove that a connected, locally path-connected space is path-connected.

21. (a) Show that every regular, second countable space is normal.

    (b) Show that every continuous, injection from a compact space into a Hausdorff space is an embedding.

22. Show that every regular Lindeloff space is normal.

(3 × 5 = 15)