QP CODE: 19002354

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Reg No : ....

Name : ....

M.Sc. DEGREE (C.S.S) EXAMINATION, NOVEMBER 2019

First Semester

Faculty of Science

MATHEMATICS

Core - ME010103 - BASIC TOPOLOGY

2019 Admission Onwards

28D1DA9D

Time: 3 Hours

Maximum Weight :30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Show that every metric space is Hausdorff
2. How will you compare two topologies on the same set using their corresponding bases?
3. Show that subspace of a discrete space is discrete
4. ***Let A be a subset of a topological space X. Then show that*** *(i)* $\bar{\phi} = \phi$, *(ii)* $\bar{\bar{A}} = \bar{A}$.
5. Define continuous function. Show that any function from a discrete topological space is continuous.
6. Define weak topology determined by the family of functions $\{f_i : X \to Y_i / i \in I\}$.
7. Define weakly hereditary property .Explain with an example
8. Let $\mathscr{C}$ be a collection of connected subsets of a space X and suppose K is a connected subset of X such that $C \cap K \neq \emptyset, \forall C \in \mathscr{C}$. Prove that $(\cup_{C\in\mathscr{C}} C) \cup K$ is connected.
9. Show that T1 property is hereditary
10. Define a regular space and hence show that every indiscrete space is regular

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Explain convergence of a sequence in a cofinite topological space

12. Prove that any intersection of an indexed family of topologies on the same set X is again a topology on X and it is the weaker than any of the topology in the family

13. Prove that a function $e : X \to Y$ is an embedding if and only if it is continuous and one to one and for every open set V in X, there exist an open subset W of Y such that $e(V) = W \cap e(X)$.

14. Define quotient map. Show that every open surjective map is a quotient map.

15. Prove that every co-countable space is Lindeloff.

16. Prove that the property of being a compact space is preserved under continuous functions.

17. Show that path connectedness is preserved under continuous onto functions

18. Show that in a normal space any closed-open inclusion contains an open-closed inclusion

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any ***two*** *questions.*

Weight ***5*** *each.*

19. *Show that the product topology on the n^{th} power of real line coincide with the usual topology* on R^n

20. Let X be a set and suppose for each x $\in$ X, a non empty family $\mathfrak{N}_x$ of subsets of X satisfying (i) if $U \in \mathfrak{N}_x$ then $x \in U$, (ii) for any $U, V \in \mathfrak{N}_x$ then $U \cap V \in \mathfrak{N}_x$, (iii) if $V \in \mathfrak{N}_x$ and $V \subseteq U$ then $U \in \mathfrak{N}_x$ (iv) if $U \in \mathfrak{N}_x$ then there exist $V \in \mathfrak{N}_x$ such that $V \subseteq U$ and $V \in \mathfrak{N}_y$ for all $y \in V$. Then show that there exist a unique topology $\mathfrak{I}$ on X such that for each x $\in$ X, $\mathfrak{N}_x$ coincides with the family of all neighborhoods of x w.r.to $\mathfrak{I}$.

21. (a)Prove that a subset of $\mathbb{R}$ is connected if and only if it is an interval.
(b)Provethat every closed and bounded interval is compact.

22. What are the equivalent conditions of locally connected space? Explain.

(2×5=10 weightage)