



QP CODE: 21000381



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Reg No :

Name :

M Sc DEGREE (CSS) EXAMINATION, MARCH 2021

Third Semester

Faculty of Science

CORE - ME010301 - ADVANCED COMPLEX ANALYSIS

M Sc MATHEMATICS, M Sc MATHEMATICS (SF)

2019 Admission Onwards

20025F71

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

Weight 1 each.

1. Prove that $\log r$ is a harmonic function.
2. Show that Poisson integral is a linear functional.
3. Find the Laurent series expansion of $f(z) = \frac{1}{z(z-1)^2}$ about the point $z = 1$.
4. Find the poles of $\frac{\pi^2}{\sin^2 \pi z}$ and the corresponding singular parts.
5. Prove that $\Gamma(z+1) = z\Gamma(z)$.
6. Prove that the zeta function can be extended to a meromorphic function in the whole plane whose only pole is a simple pole at $s=1$ with residue 1.
7. Prove that $\xi(s) = \frac{1}{2}s(1-s)\pi^{-\left(\frac{s}{2}\right)}\Gamma\left(\frac{s}{2}\right)\zeta(s)$ is entire.
8. What do you mean by the trivial zeros of the Riemann zeta function?
9. Define the Riemann mapping.
10. Define a free boundary arc. Give an example.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

Weight 2 each.





11. Let Ω be a symmetric region. Prove that $u(z)$ and $u(\bar{z})$ are simultaneously harmonic in Ω . Also prove that $f(z)$ and $f(\bar{z})$ are simultaneously analytic in Ω .
12. Prove that a continuous function $U(z)$, which satisfies the mean value property is necessarily harmonic.
13. Prove that the infinite product $\prod_1^\infty (1 + a_n)$ with $(1 + a_n) \neq 0$ converges simultaneously with the series $\sum_1^n \log(1 + a_n)$ whose terms represent the values of the principal branch of the logarithm.
14. State and prove Poisson-Jensen's formula.
15. State and prove a characterisation theorem for a family \mathcal{F} of functions to be normal.
16. If a family \mathcal{F} of continuous functions with values in a metric space S is equicontinuous on every compact subsets of Ω and for any $z \in \Omega$, $f \in \mathcal{F}$, the values $f(z)$ lie in a compact subset of S , then prove that \mathcal{F} is normal in Ω .
17. Prove that any two bases of the period module are connected by a unimodular transformation.
18. Define the ζ - function. Prove that $\zeta(z) = \frac{1}{z} + \sum_{\omega \neq 0} \left(\frac{1}{z - \omega} + \frac{1}{\omega} + \frac{z}{\omega^2} \right)$

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. Derive any four properties of subharmonic functions.
20. (i) Derive the Taylor series for an analytic function $f(z)$ in a region Ω containing the point z_0 .
(ii) State and prove Weirstrass's theorem for the convergence of a sequence of analytic functions.
(iii) State and prove Hurwitz theorem.
21. (i) Prove that the Riemann Zeta function is analytic in the half plane $Re s > 1$.
(ii) Prove that for $\sigma > 1$, $\zeta(s) = \prod_{p_n, prime} \left(\frac{1}{1 - p_n^s} \right)^{-1}$.
(iii) Prove that for $\sigma > 1$, $\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^x - 1} dx$.
22. Prove that $[\wp'(z)]^2 = 4[\wp(z)]^3 - g_2\wp(z) - g_3$, where g_2 and g_3 are constants.

(2×5=10 weightage)

