21101717

QP CODE: 21101717

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) SPECIAL SUPPLEMENTARY EXAMINATION, JULY 2021

Fifth Semester

CORE COURSE - MM5CRT03 - ABSTRACT ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2018 Admission Only

5A099B9E

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Check whether the usual multiplication is a binary operation on the set $\mathbb{Z}^+$.
2. Define order of a group.
3. Define greatest common divisor of two positive integers r and s.
4. Define the n**th dihedral group**. Give the elements of the third dihedral group.
5. Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$ in S_8 as a product of transpositions.
6. Show that if σ is a cycle of odd length, then σ^2 is a cycle.
7. Define the **alternating group** A_n **on** n **letters**. What is its order?
8. Check whether $\phi : (M_n(\mathbb{R}), +) \to (\mathbb{R}, +)$ defined by $\phi(A) =$ trace of A is a group homomorphism or not.
9. Define inner automorphism.
10. Define a) ring homomorphism b) ring isomorphism

11. Find all solutions of the equation $x^2 + 2x + 2 = 0$ in Z_6

12. Give an example to show that a factor ring of an integral domain may be a field

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove that $\langle \mathbb{Q}^+, * \rangle$ is a group, where $*$ is defined by $a * b = ab/2$.

14. Let G be a group with binary operation $*$ .Then prove that the identity element and inverse of each element are unique in G .

15. Let G be a group and let $a \in G$. Then prove that $H = \{a^n / n \in \mathbb{Z}\}$ is a subgroup of G and is the smallest subgroup of G that contains a.

16. Let G be a group. Prove that the permutations $\rho_a : G \to G$, where $\rho_a(x) = xa$ for $a \in G$ and $x \in G$, do form a group isomorphic to G.

17. Suppose H and K are subgroups of a group G such that $K \leq H \leq G$, and suppose $(H : K)$ and $(G : H)$ are both finite. Then prove that $(G : K)$ is finite, and $(G : K) = (G : H)(H : K)$.

18. If $\phi : G \to G'$ is a group homomorphism, show that ϕ is one to one if and only if $ker\,\phi$ is trivial.

19. Let a group **G** contains a nontrivial sbgroup of index 2. Show that **G** is not simple.

20. a) Mark each of the following true or false.

    i) Every field is an integral domain

    ii) The characteristic of nZ is n

    b) Prove that Z_p is a field if p is a prime.

21. Prove that a ring homomorphism $\phi : R \to R'$ is a one to one map if and only if $\ker \phi = \{0\}$

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Define isomorphism between two binary structures. Check whether $\langle \mathbb{Z}, + \rangle$ is isomorphic to $\langle 2\mathbb{Z}, + \rangle$ where $+$ is the usual addition.

23.

1. Let H be a subgroup of a group G. Let the relation $\sim_L$ be defined on G by $a \sim_L b$ if and only if $a^{-1}b \in H$. Then show that $\sim_L$ is an equivalence relation on G. What is the cell in the corresponding partition of G containing $a \in G$?
2. Let H be the subgroup $< \mu_1 >= \{\rho_0, \mu_1\}$ of S_3. Find the partitions of S_3 into left cosets of H, and the partition into right cosets of H.

24. Let H be a subgroup of a group G. prove that $aHbH = abH$ defines a binary operation on G/H if and only if H is a normal subgroup of G. Then furthere show that if H is a normal subgroup of a group G then G/H is a group. under the binary operation $aHbH = abH$.

25. a) Let p be a prime . Show that in a ring Z_p , $(a + b)^p = a^p + b^p$ for all $a, b \in Z_p$

b) Show that if a and b are nilpotent elements of a commutative ring , then a + b is also nilpotent.

c) Show that intersection of subrings of a ring R is again a subring of R

(2×15=30)